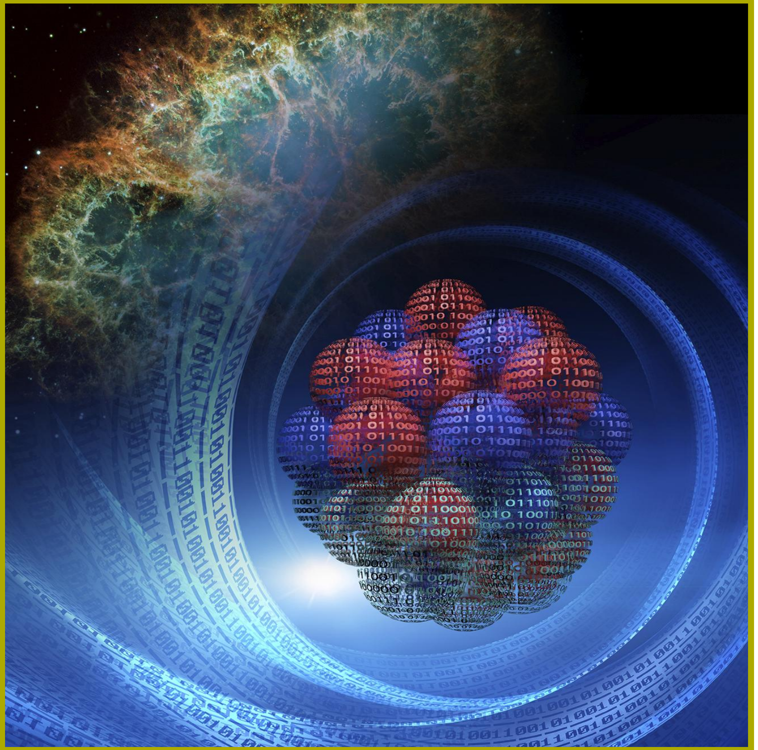
WOUTER RYSSENS



Andy Sproles, ORNL

Microscopic models of nuclear structure at scale

W. Ryssens

19th of March 2025

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BRUSSELS LABORATORY OF THE UNIVERSE

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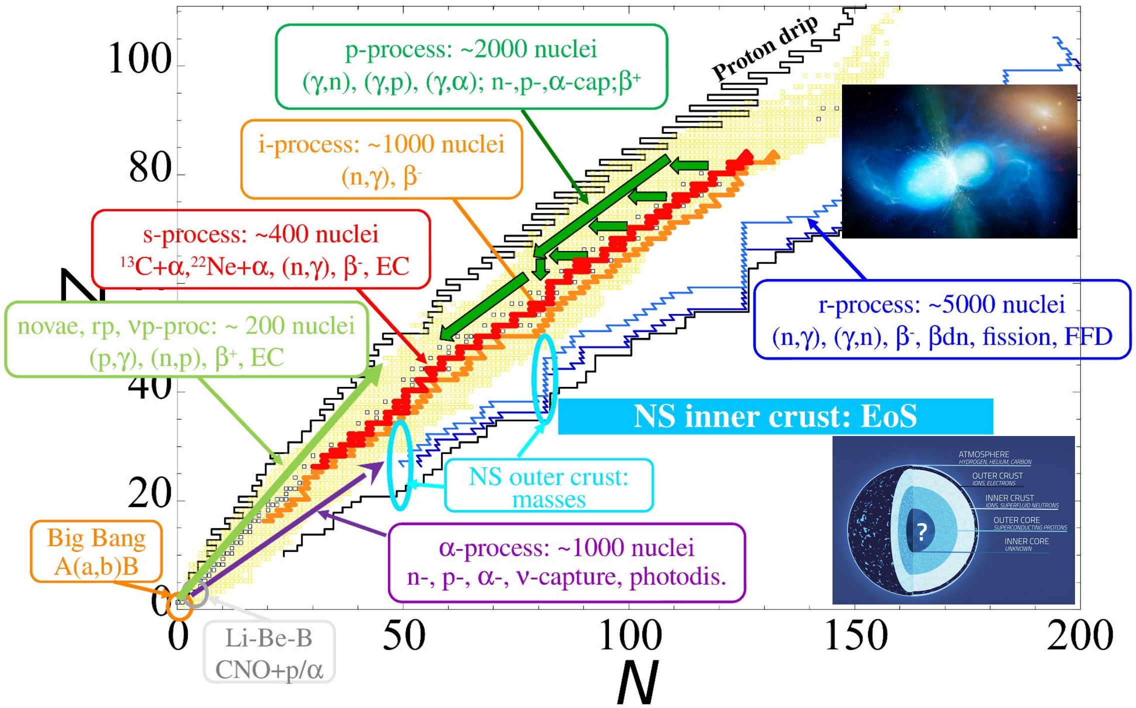
The nuclear chart and the processes traversing it

Extrapolations in
- nucleon number
- energy
- temperature
- density
- ……



The nuclear chart and the processes traversing it

Extrapolations in
- nucleon number
- energy
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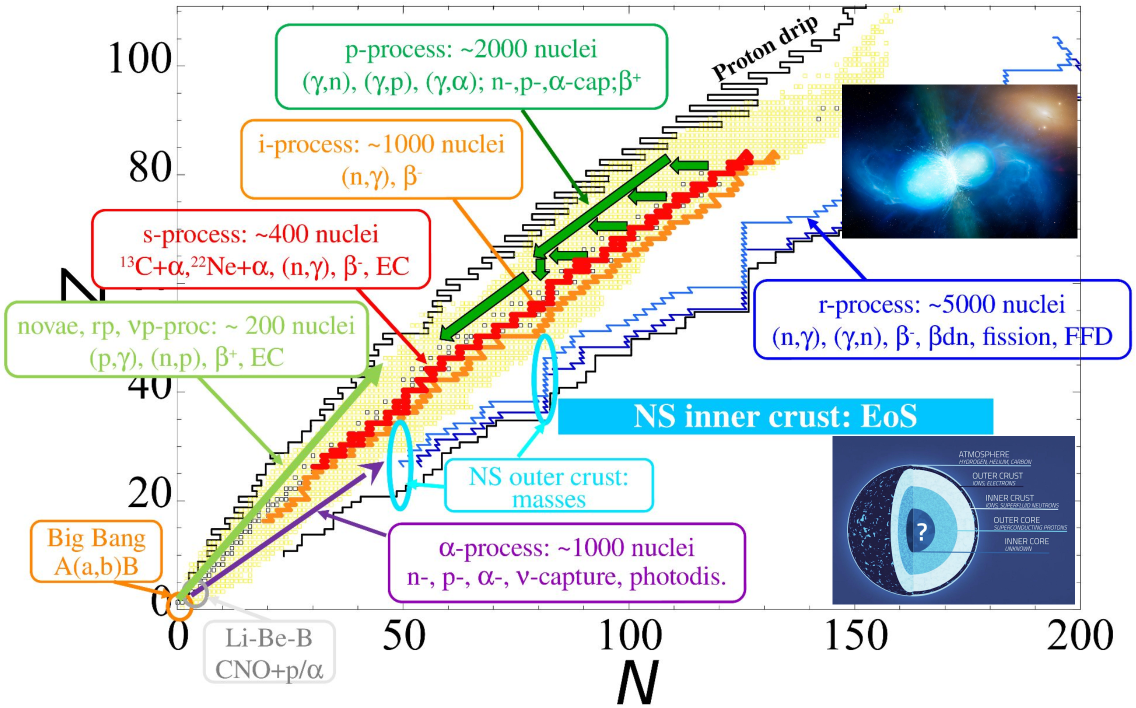
- ……

and all of that for
- **~7000** nuclei
- **many** reactions



The nuclear chart and the processes traversing it

Extrapolations in
- nucleon number
- energy
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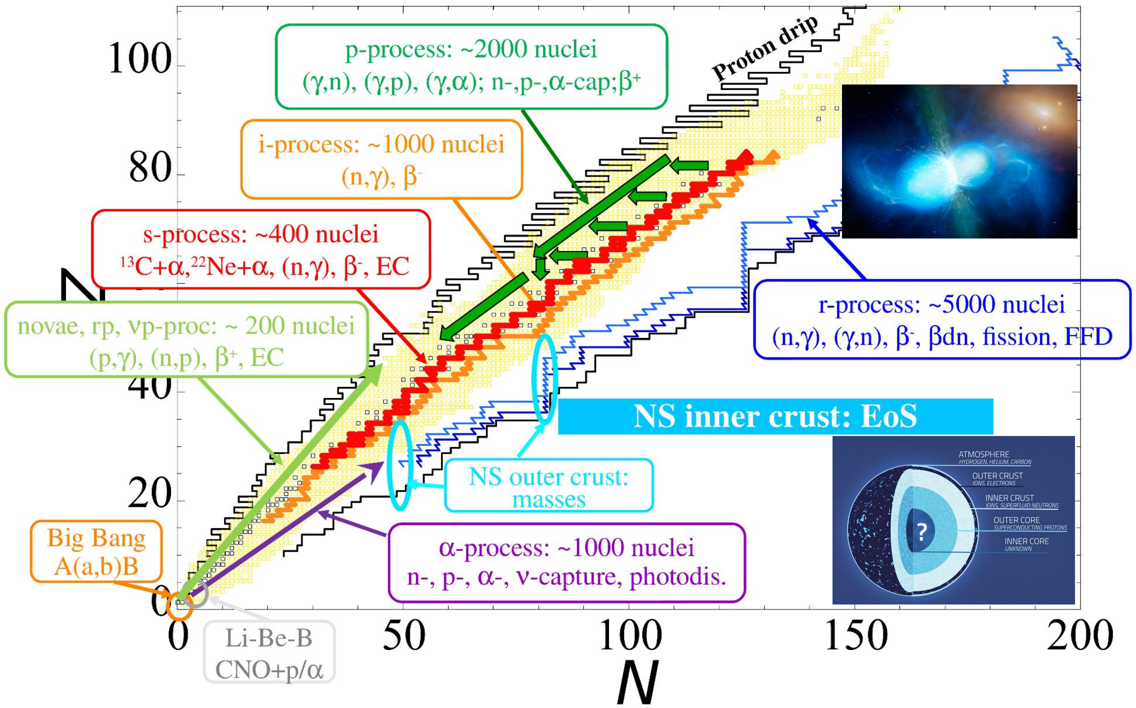
- ……

and all of that for
- **~7000** nuclei
- **many** reactions

We need structure models that are

1. **predictive**….
2. but **complete!**



Skyrme **E**nergy **D**ensity **F**unctionals (**EDF**s)

$$E \sim \int d^3r \Big[C^\rho \rho(\mathbf{r})\rho(\mathbf{r}) + C^\tau \tau(\mathbf{r})\rho(\mathbf{r}) + \dots \Big]$$

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$$E \sim \int d^3r \left[C^{\rho} \boxed{\rho(\mathbf{r})\rho(\mathbf{r})} + C^{\tau} \boxed{\tau(\mathbf{r})\rho(\mathbf{r})} + \ldots \right]$$

Local densities and currents of a wavefunction

Skyrme **E**nergy **D**ensity **F**unctionals (**EDF**s)

Coupling constants (~ **25** parameters) fitted to data

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Local densities and currents of a wavefunction

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Local densities and currents of a wavefunction

Progress?

1. more physics in the wavefunction

Skyrme **E**nergy **D**ensity **F**unctionals (**EDF**s)

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Local densities and currents of a wavefunction

Progress?

1. more physics in the wavefunction
2. more (pseudo-) observables simultaneously
   "more" = both **scale** and **diversity**!

Skyrme **E**nergy **D**ensity **F**unctionals (**EDF**s)

Energy

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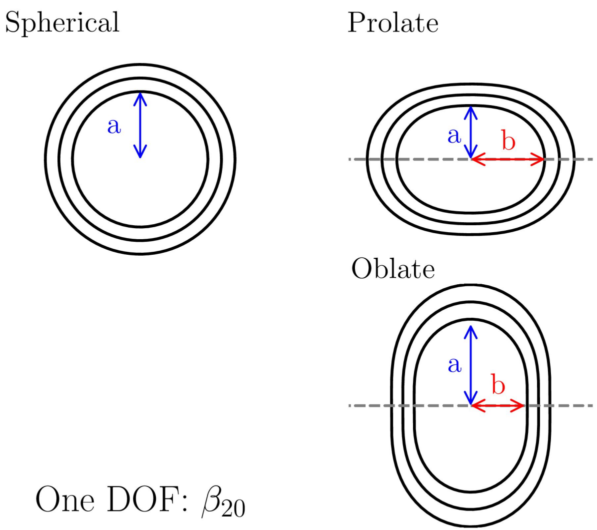
$$\boxed{E} \sim \int d^3r \left[\boxed{C^\rho} \boxed{\rho(\mathbf{r})\rho(\mathbf{r})} + \boxed{C^\tau} \boxed{\tau(\mathbf{r})\rho(\mathbf{r})} + \dots \right]$$

Local densities and currents of a wavefunction

Progress?

1. more physics in the wavefunction
2. more (pseudo-) observables simultaneously
   “more” = both **scale** and **diversity**!
3. search for a “better” EDF form

Large-scale models in 1-2 dimensions



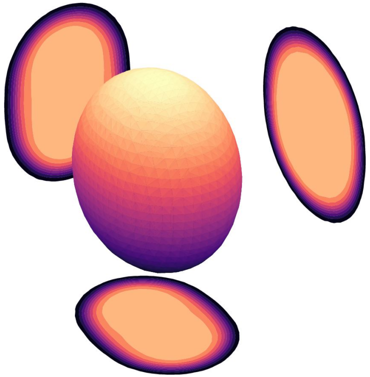
One DOF: β_{20}

Nuclear deformation

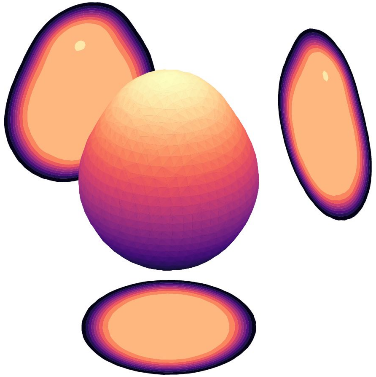
- larger variational space
- shape DOF characterized by multipole moments
- capture correlations at modest CPU cost
- intuitive interpretation

Large-scale models in 1-2-**3** dimensions

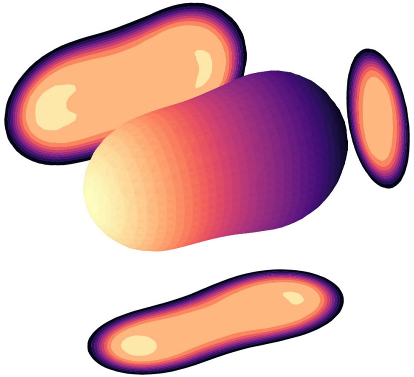
β_{20}, β_{22} or β_2, γ



β_{20}, β_{30}



β_{20}, β_{22} **and** β_{30}



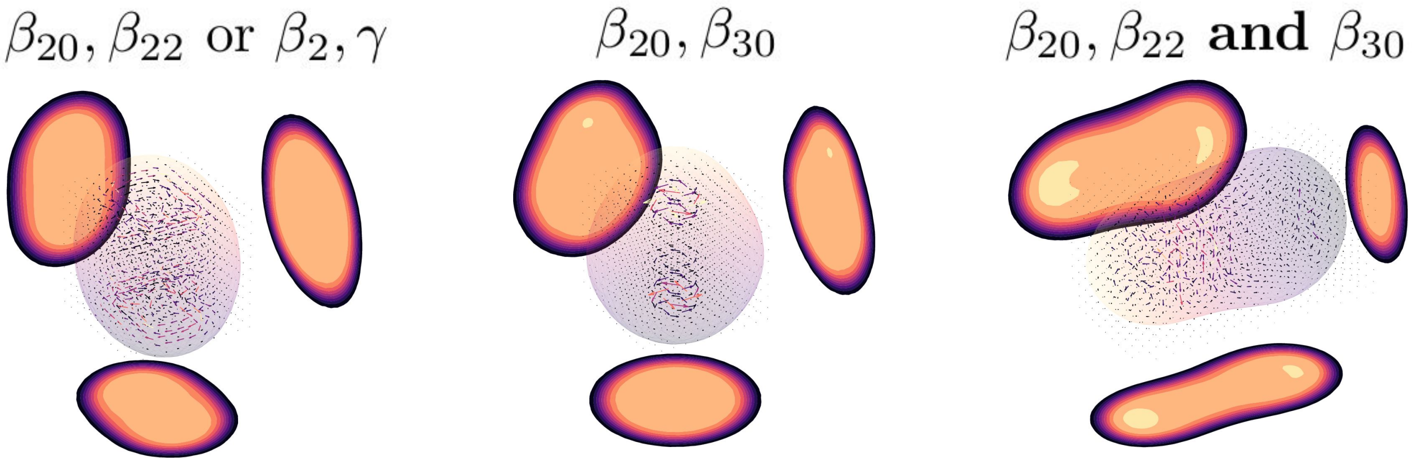
Nuclear deformation

- larger variational space
- shape DOF characterized by multipole moments
- capture correlations at modest CPU cost
- intuitive interpretation

More general configurations

- triaxial shapes
- reflection asymmetry
- elongated shapes

Large-scale models in 1-2-**3** dimensions



Nuclear deformation

- larger variational space
- shape DOF characterized by multipole moments
- capture correlations at modest CPU cost
- intuitive interpretation

More general configurations

- triaxial shapes
- reflection asymmetry
- elongated shapes
- spin densities and currents

Brussels-**Sk**yrme-on-a-**G**rid: BSkG

BSkG1: G. Scamps et al., EPJA **57**, 333 (2021).
BSkG2: W. Ryssens et al., EPJA **58**, 246 (2022).
W. Ryssens et al., EPJA **59**, 96 (2023).
BSkG3: G. Grams et al., EPJA **59**, 270 (2023).
BSkG4: G. Grams et al., arXiv:2411.08007 (2024).

BSkG1

- fitted to 2457 masses
- fitted to 884 charge radii
- includes triaxial deformation

Rms σ	BSkG1	BSkG2	BSkG3	BSkG4
Masses [MeV]	0.741			
S_n [MeV]	0.466			
Radii [fm]	0.024			
Prim. barriers [MeV]				
Sec. barriers [MeV]				
Fission isomers [MeV]				
Max. NS mass [$M_\odot$]				

Brussels-**Sk**yrme-on-a-**G**rid: BSkG

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BSkG1

- fitted to 2457 masses
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Rms σ	BSkG1	BSkG2	BSkG3	BSkG4
Masses [MeV]	0.741	0.678		
S_n [MeV]	0.466	0.500		
Radii [fm]	0.024	0.027		
Prim. barriers [MeV]	0.88	0.44		
Sec. barriers [MeV]	0.87	0.47		
Fission isomers [MeV]	1.0	0.49		
Max. NS mass [$M_\odot$]				

BSkG2

- <u>**complete**</u> time-reversal breaking
- fit to 45 reference fission barriers
  + 28 fission isomers

Brussels-**Sk**yrme-on-a-**G**rid: BSkG

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BSkG1

- fitted to 2457 masses
- fitted to 884 charge radii
- includes triaxial deformation

Rms σ	BSkG1	BSkG2	BSkG3	BSkG4
Masses [MeV]	0.741	0.678	0.631	
S_n [MeV]	0.466	0.500	0.442	
Radii [fm]	0.024	0.027	0.024	
Prim. barriers [MeV]	0.88	0.44	0.33	
Sec. barriers [MeV]	0.87	0.47	0.51	
Fission isomers [MeV]	1.0	0.49	0.34	
Max. NS mass [$M_\odot$]	1.8	1.8	2.3	

BSkG2

- <u>**complete**</u> time-reversal breaking
- fit to 45 reference fission barriers
  + 28 fission isomers

BSkG3

- extended Skyrme EDF form
- supports massive neutron stars

Brussels-**Sk**yrme-on-a-**G**rid: BSkG

BSkG1: G. Scamps et al., EPJA **57**, 333 (2021).
BSkG2: W. Ryssens et al., EPJA **58**, 246 (2022).
W. Ryssens et al., EPJA **59**, 96 (2023).
BSkG3: G. Grams et al., EPJA **59**, 270 (2023).
BSkG4: G. Grams et al., EPJA **61,** 35 (2024).

BSkG1

- fitted to 2457 masses
- fitted to 884 charge radii
- includes triaxial deformation

BSkG4

- pairing reproduces advanced INM calculations
- ideal for TD simulations in NS crust

Rms σ	BSkG1	BSkG2	BSkG3	BSkG4
Masses [MeV]	0.741	0.678	0.631	0.633
S_n [MeV]	0.466	0.500	0.442	0.402
Radii [fm]	0.024	0.027	0.024	0.025
Prim. barriers [MeV]	0.88	0.44	0.33	0.36
Sec. barriers [MeV]	0.87	0.47	0.51	0.53
Fission isomers [MeV]	1.0	0.49	0.34	0.33
Max. NS mass [$M_\odot$]	1.8	1.8	2.3	2.3

BSkG2

- <u>**complete**</u> time-reversal breaking
- fit to 45 reference fission barriers + 28 fission isomers

BSkG3

- extended Skyrme EDF form
- supports massive neutron stars

Investment in computing

MOCCa

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Density functional theory with MOCCa

- nonlinear optimisation problem
- algorithms for speed and robustness
- open-source later this year

Investment in computing

MOCCa

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    '.___.'
```

Density functional theory with MOCCa

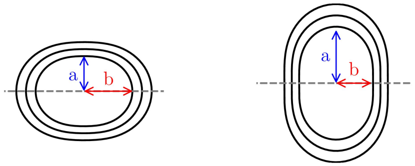
- nonlinear optimisation problem
- algorithms for speed and robustness
- open-source later this year

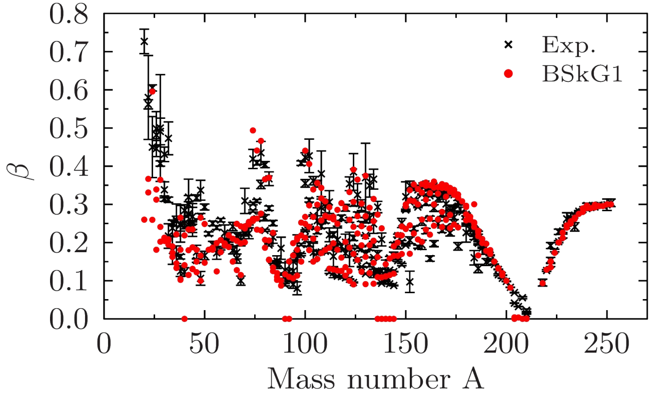
Raw computing power

- CÉCI & VSC: academic support in Belgium
- TIER-1 resources by Cenaero
  - Lucia - ranked 388th - our development workhorse
- TIER-0 resources by EuroHPC
  - MN 5 - ranked 11th - large scale fission
  - LUMI - ranked 8th - nuclear pasta

Deformations

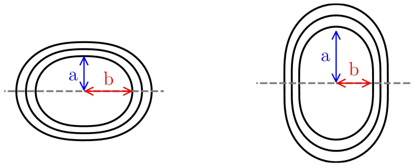
“Ordinary” quadrupole deformation

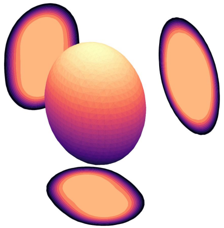


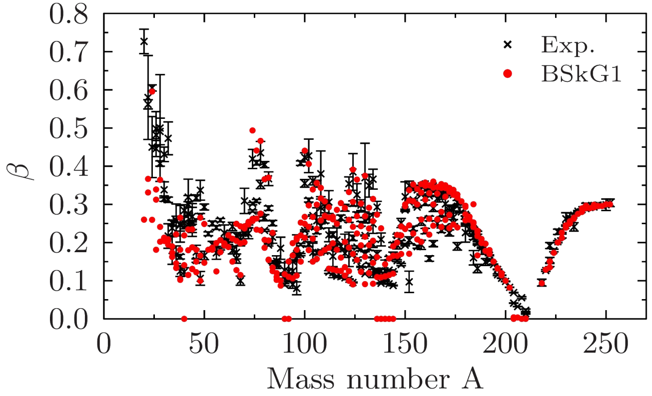


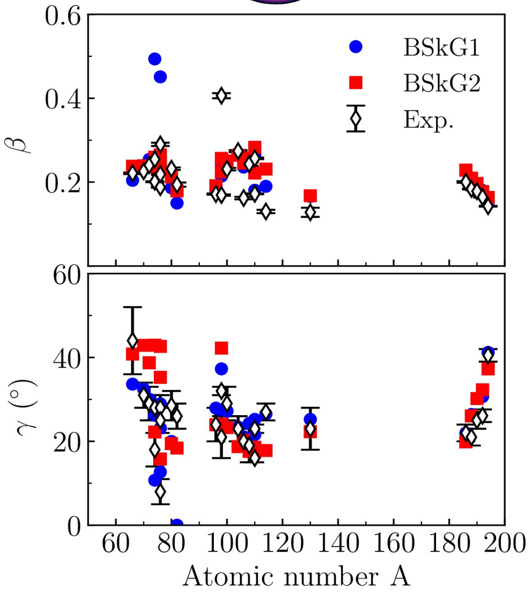
Deformations

“Ordinary” quadrupole deformation … and triaxial deformation …





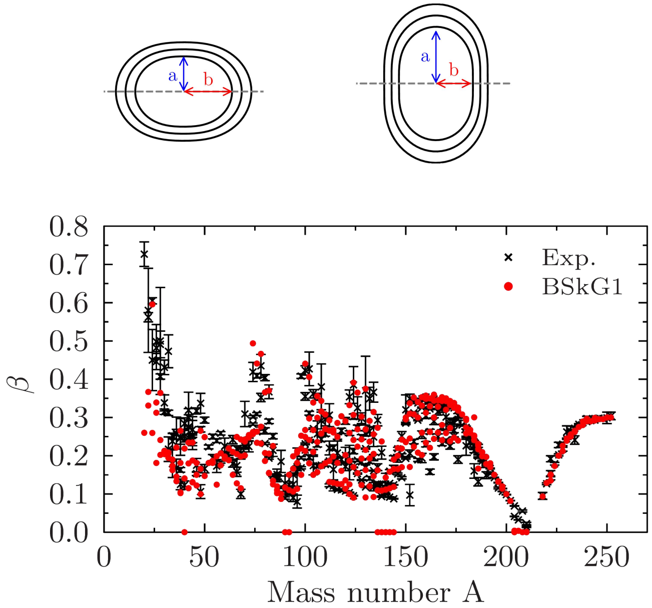




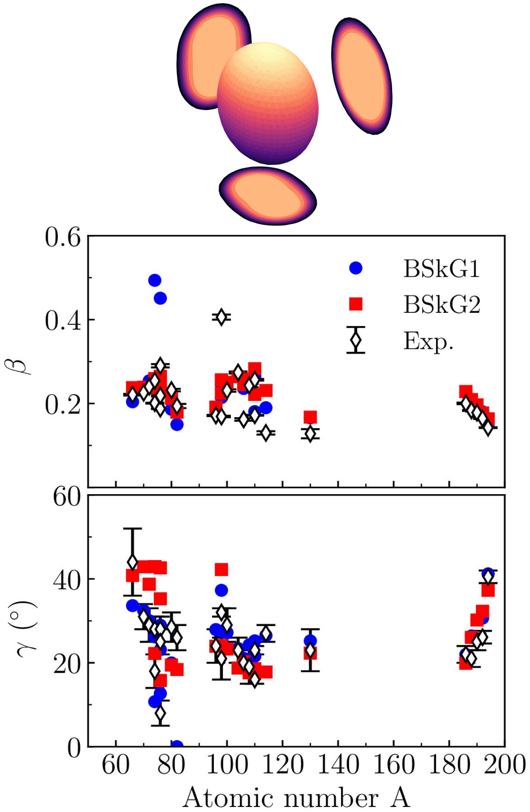
Deformations

G. Scamps et al., EPJA **57**, 333 (2021).
G. Grams et al., arXiv:2411.08007 (2024).
W.R. et al., Phys. Rev. Lett. **130**, 212302 (2024).

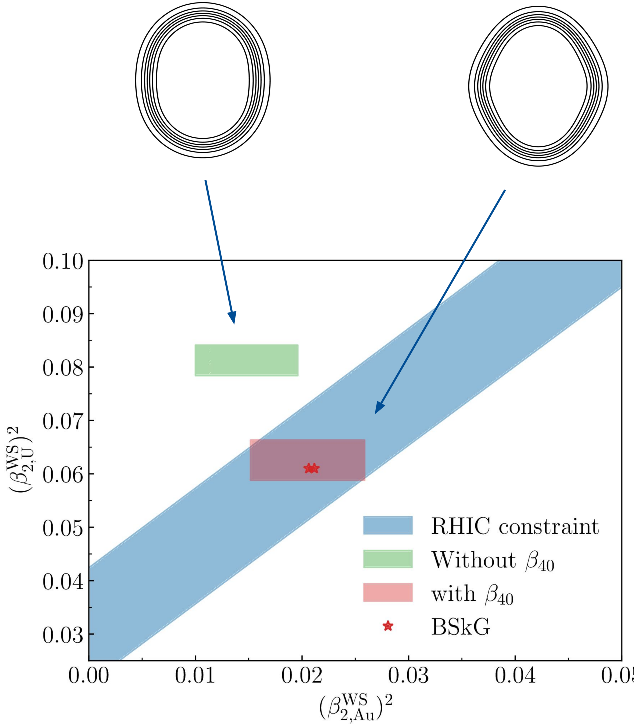
"Ordinary" quadrupole deformation



… and triaxial deformation …

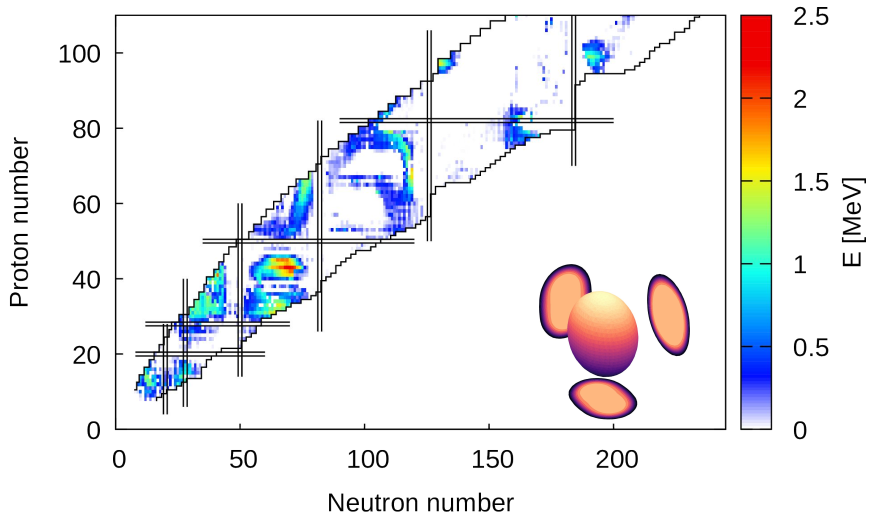


… and even hexadecapole!



Masses

G. Scamps et al., EPJA **57**, 333 (2021).
G. Grams et al., EPJA **59**, 270 (2023).



Triaxial deformation

- many nuclei are affected
- effects up to 2.5 MeV near Z~44

Masses

G. Scamps et al., EPJA **57**, 333 (2021).
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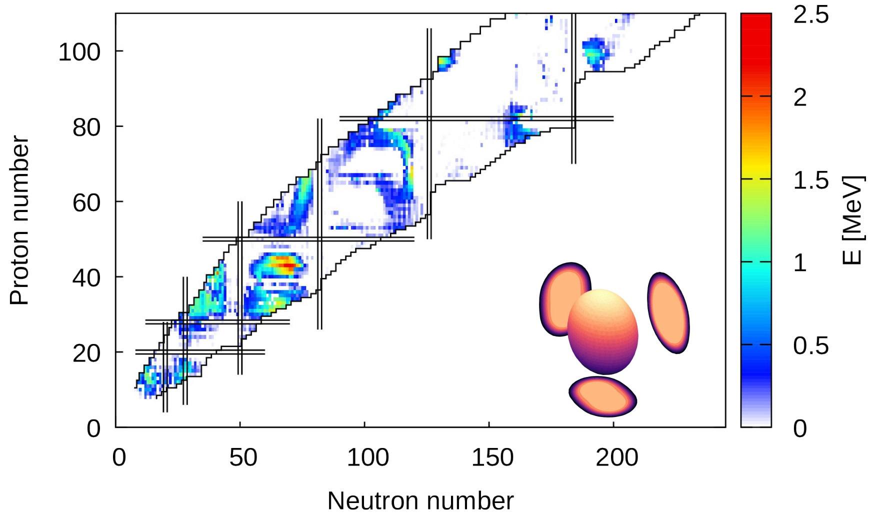
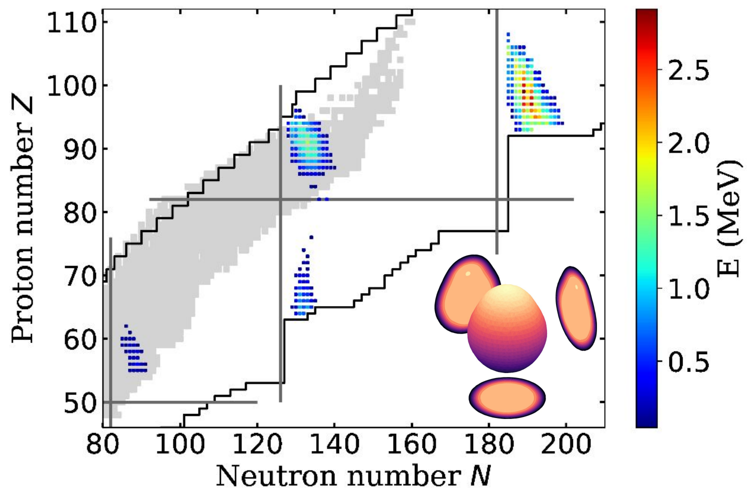




Triaxial deformation

- many nuclei are affected
- effects up to 2.5 MeV near Z~44

Reflection asymmetry

- small number of known nuclei affected
- Near N=184:
  - large effect up to 2.5 MeV
  - dripline modified
  - fission properties modified

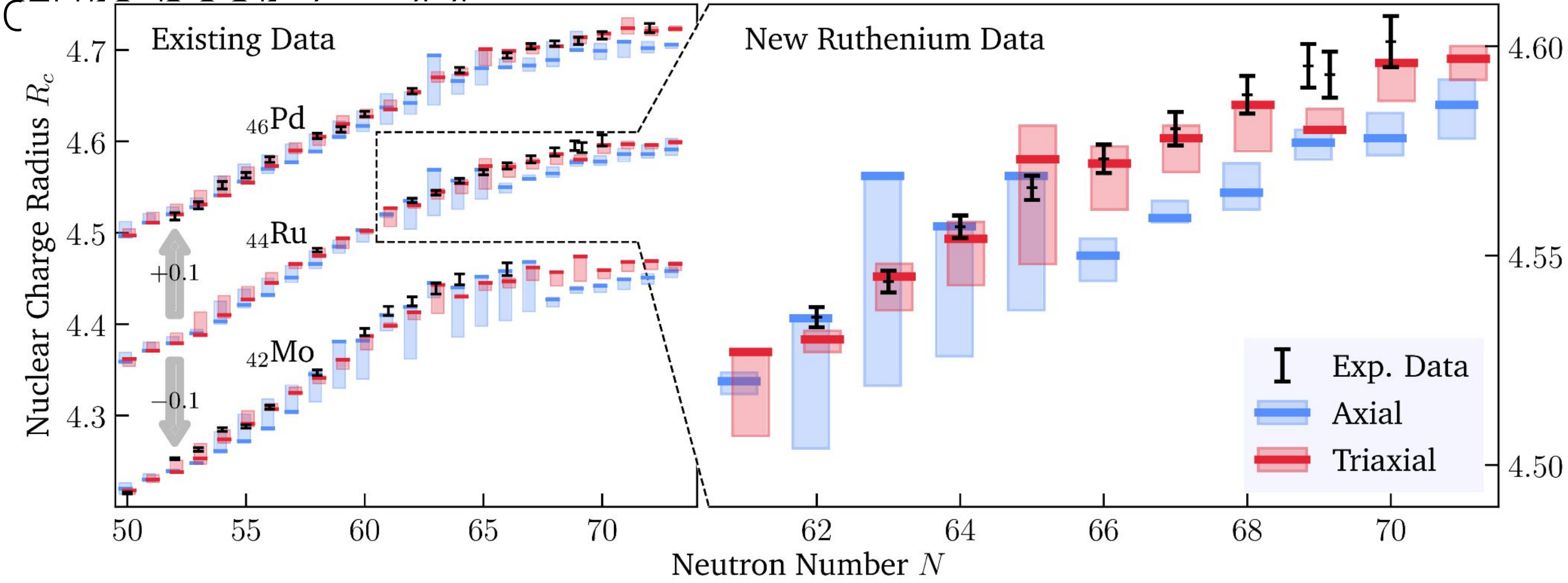
Structure near Z ~ 44

Triaxial deformation impacts

- spectroscopy
- patterns in masses
- existence and structure of isomers

M. Hukkanen, W.R. et al., PRC **107** (2023).
M. Hukkanen, W.R. et al., PRC **108** (2023).
M. Hukkanen, W.R. et al., PLB **856** (2024).
M. Stryjczyk, W. R. et al., PLB **862** (2025).
B. Maass et al.,arXiv:2503.07841 (2024).

Structure near Z = 44



Triaxial deformation impacts

- spectroscopy
- patterns in masses
- existence and structure of isomers
- ❗ charge radii

  contrary to claims in the literature!

M. Hukkanen, W.R. et al., PRC **107** (2023).
M. Hukkanen, W.R. et al., PRC **108** (2023).
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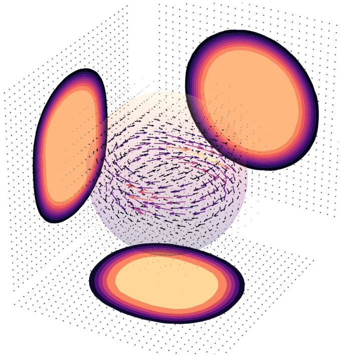
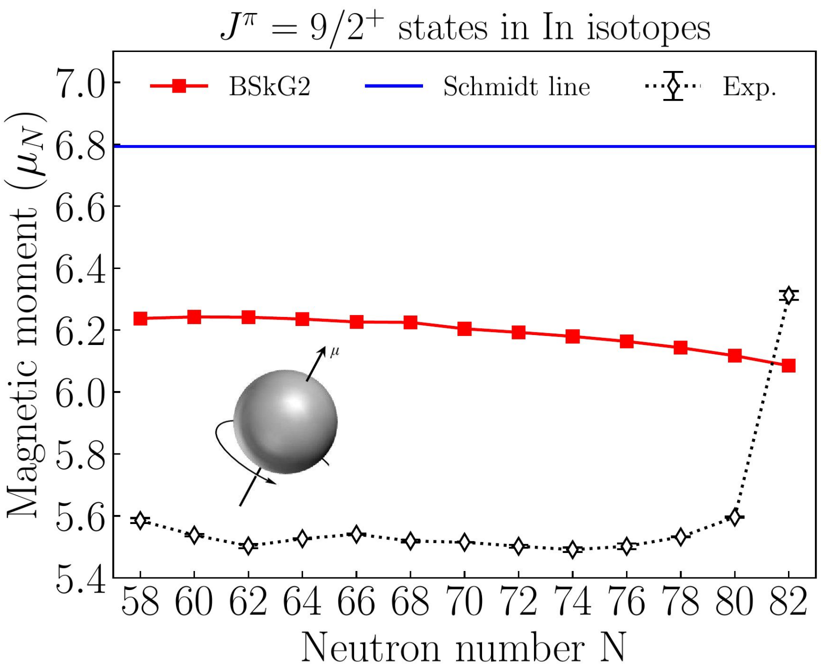
Time-odd channel





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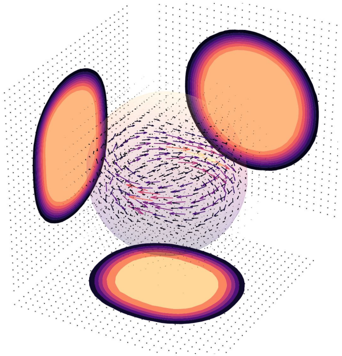
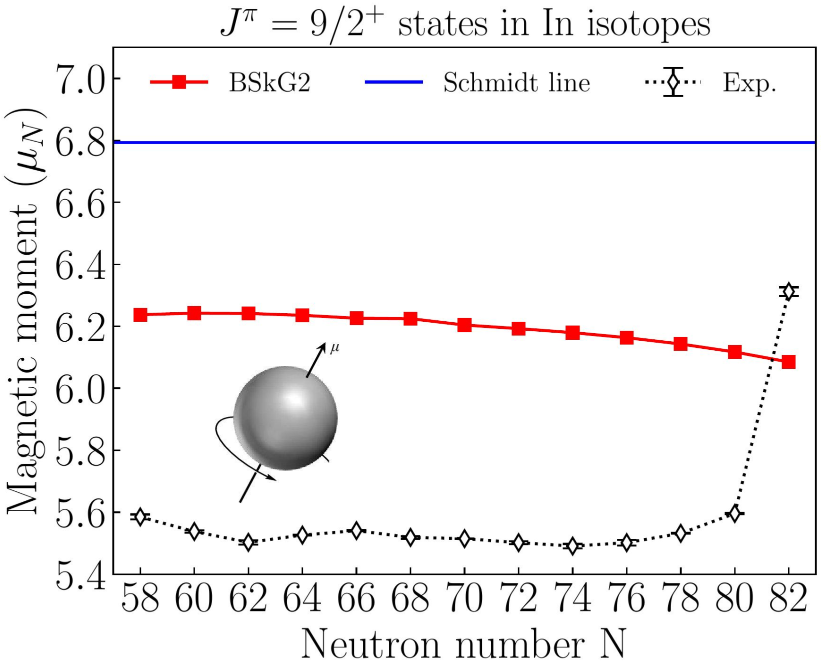
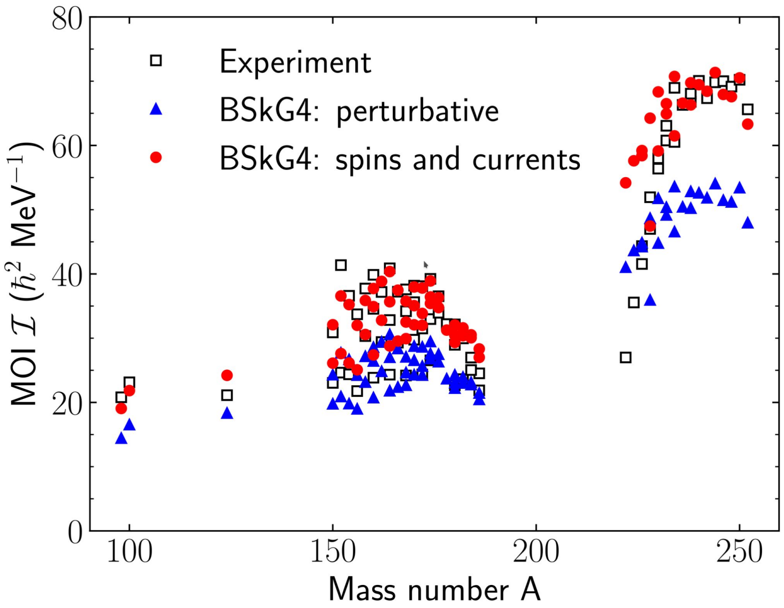
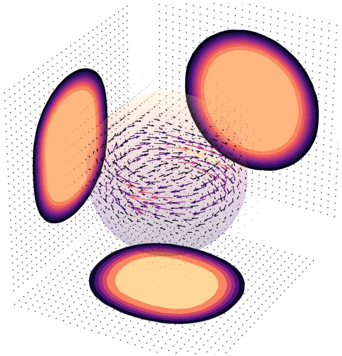


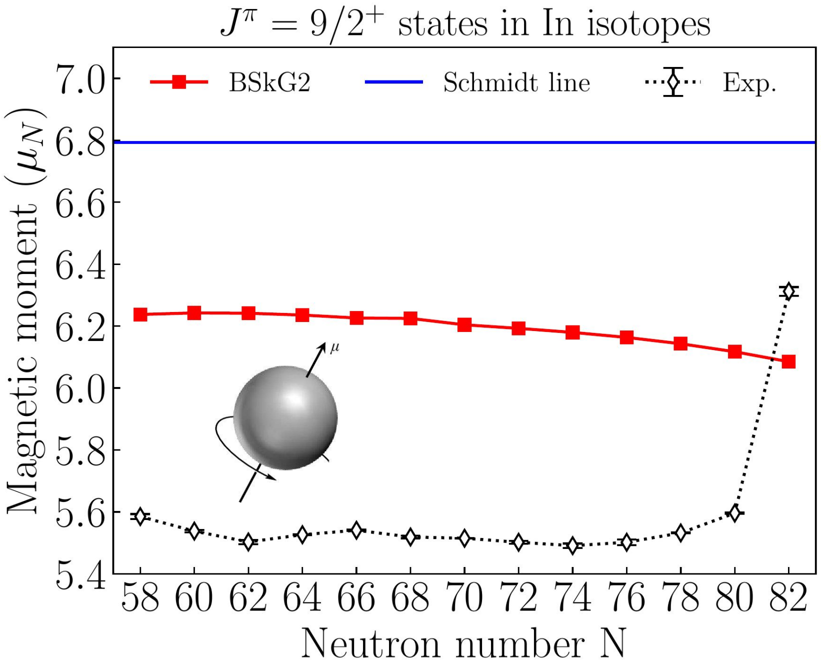


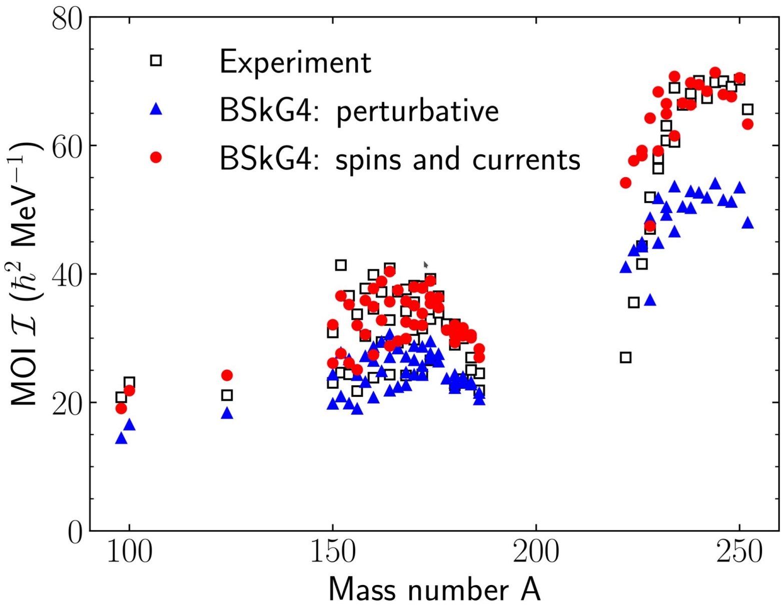


Time-odd channel

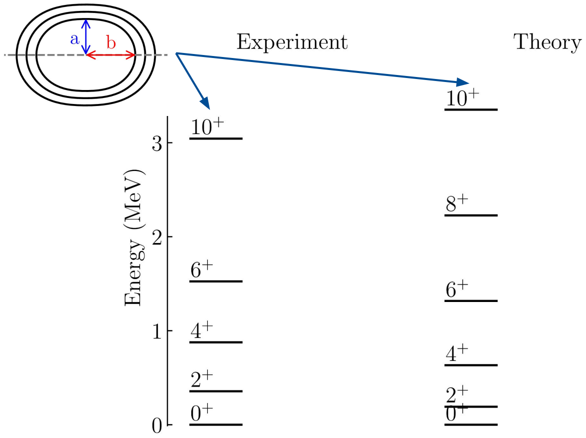
L. Gonzales-Miret: Improving treatment of collective motion







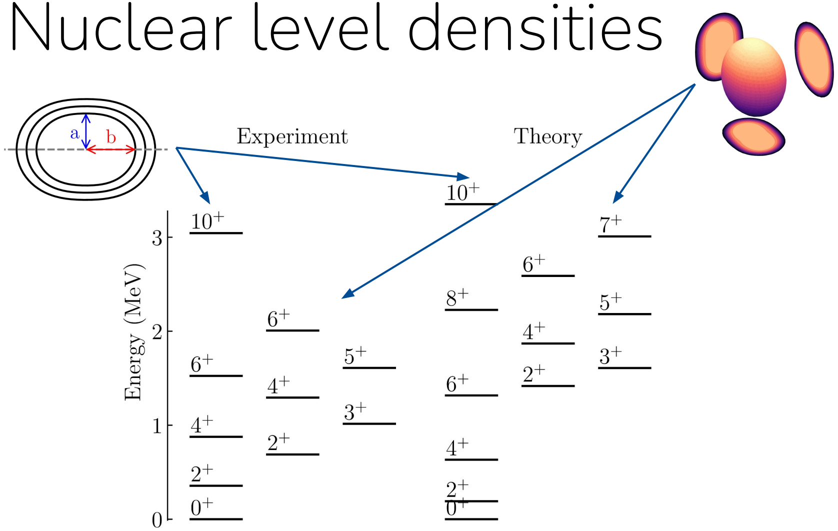
Nuclear level densities



Symmetries dictate NLDs

- larger collective enhancement

Nuclear level densities



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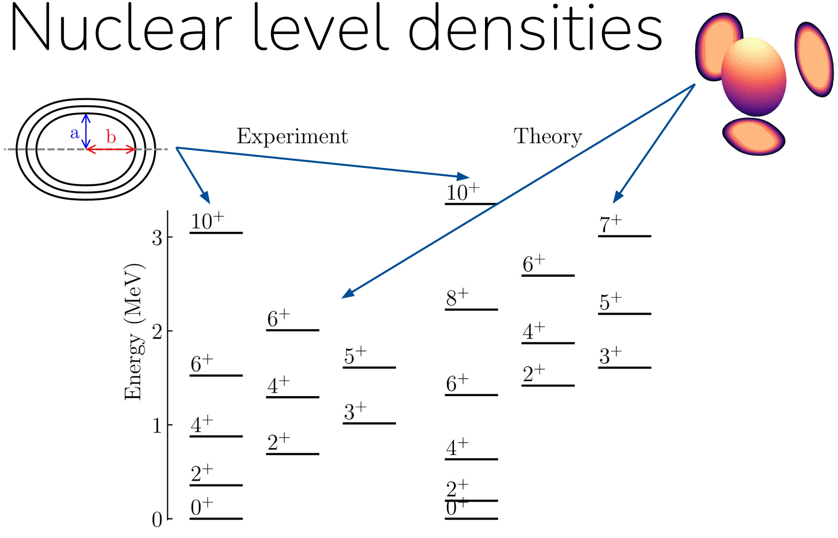
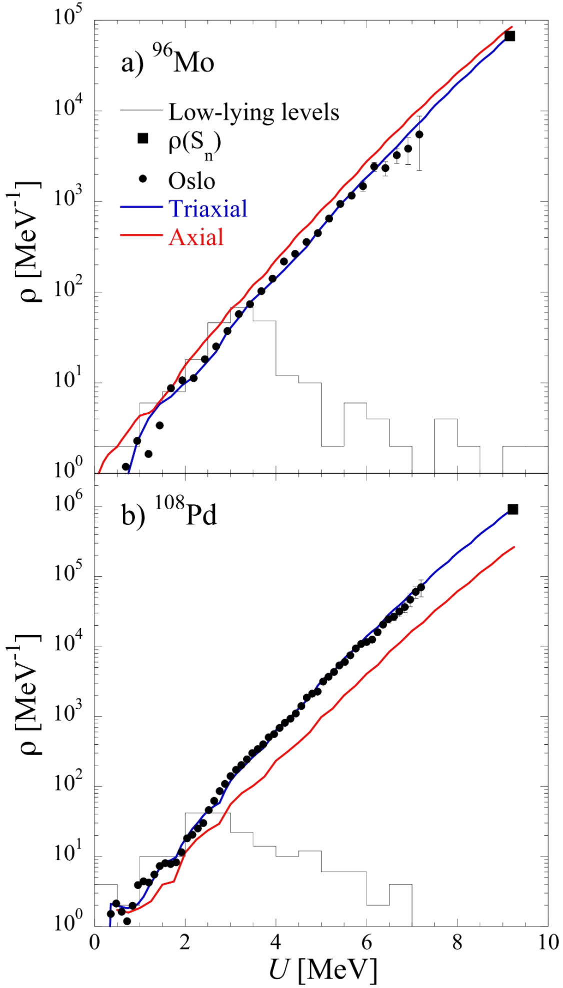
Nuclear level densities



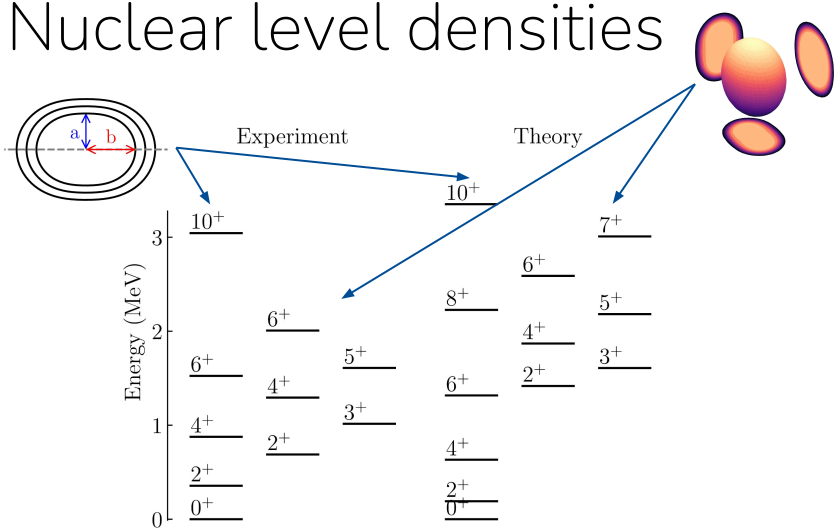
Symmetries dictate NLDs

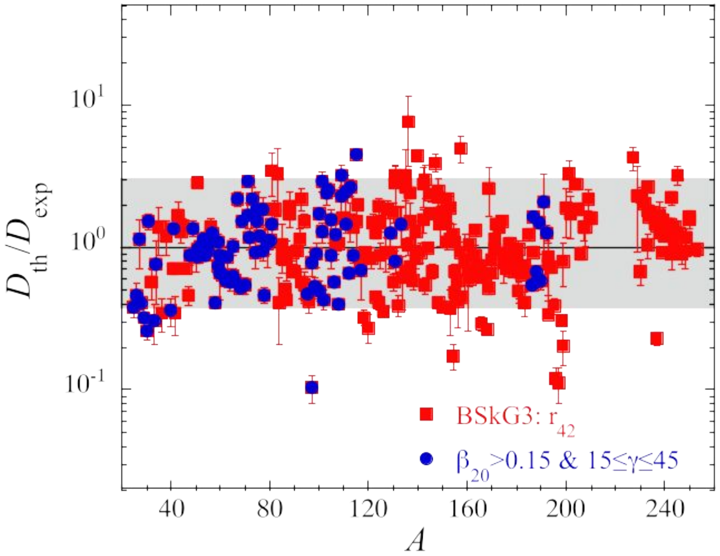
- larger collective enhancement
- combinatorial calculation of NLDs
- ... but the story is not so simple!



Nuclear level densities

S. Goriely et al., in preparation.





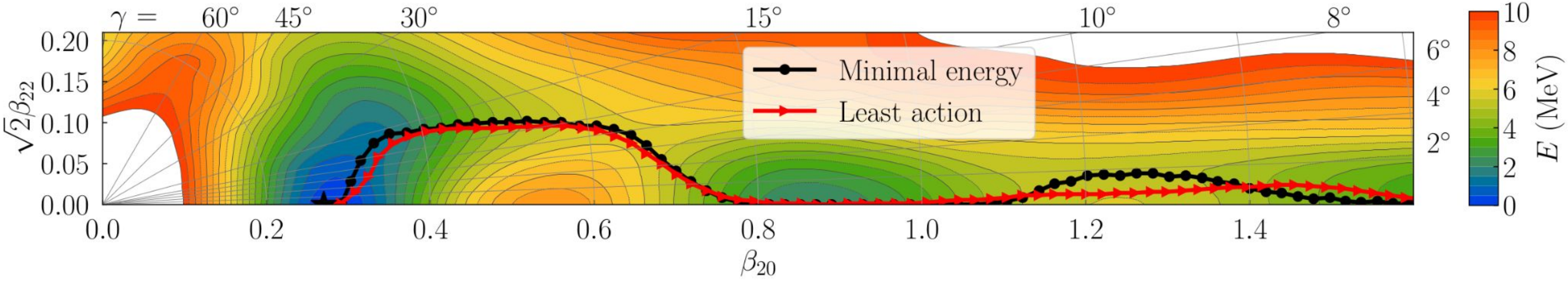
Symmetries dictate NLDs

- larger collective enhancement
- combinatorial calculation of NLDs
- ... but the story is not so simple!

$$f_{\rm rms} = \exp\left[\frac{1}{N_e}\sum_i^{N_e}(\ln r_i)^2\right]^{1/2} = 1.96\,.$$

Fission barriers

W. R. et al., EPJA **59**, 96 (2023).
E. Flynn et al., PRC **105** (2022).
S. Bara et al. & A. Sánchez-Fernández et al., in preparation.



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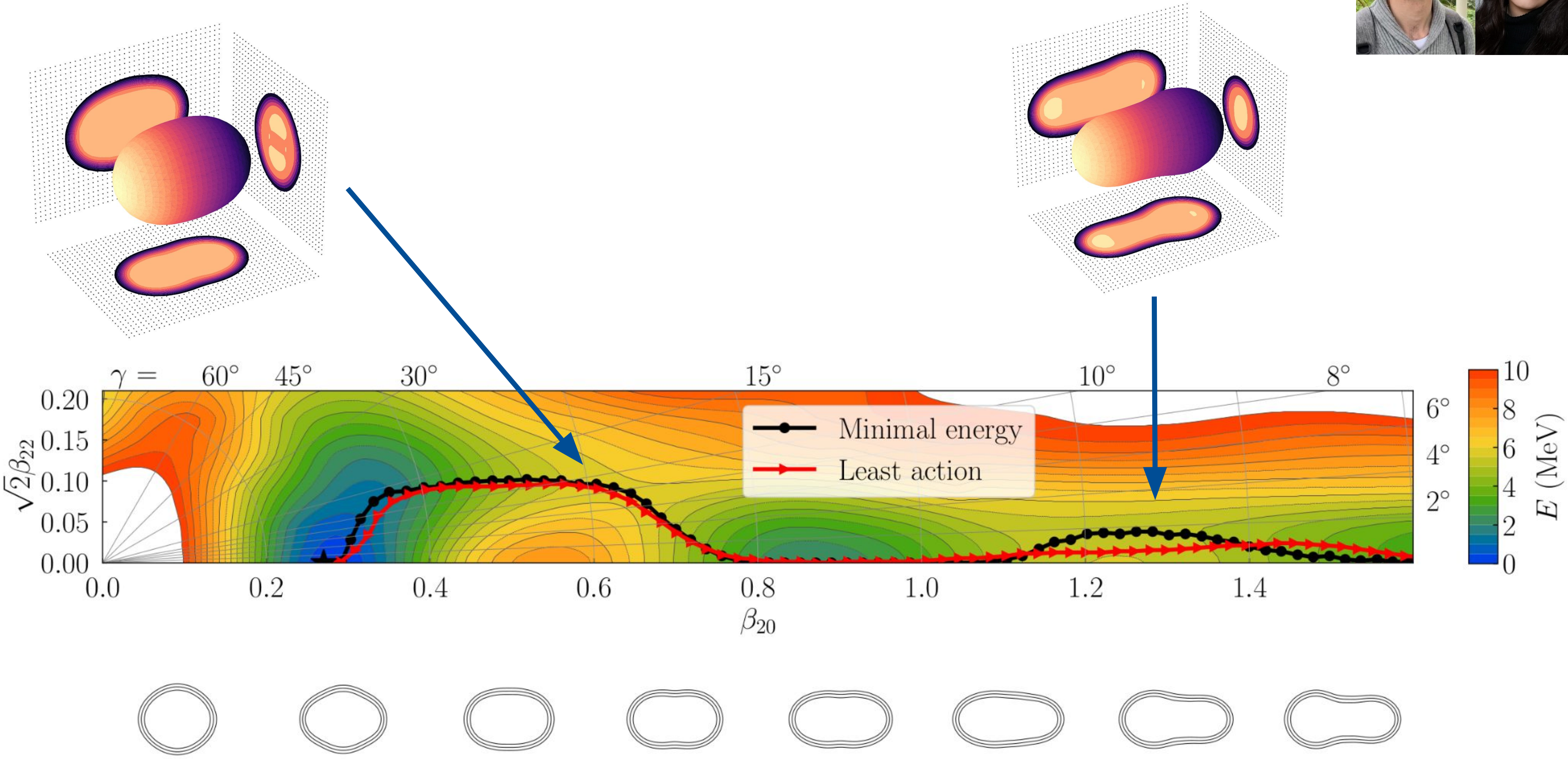
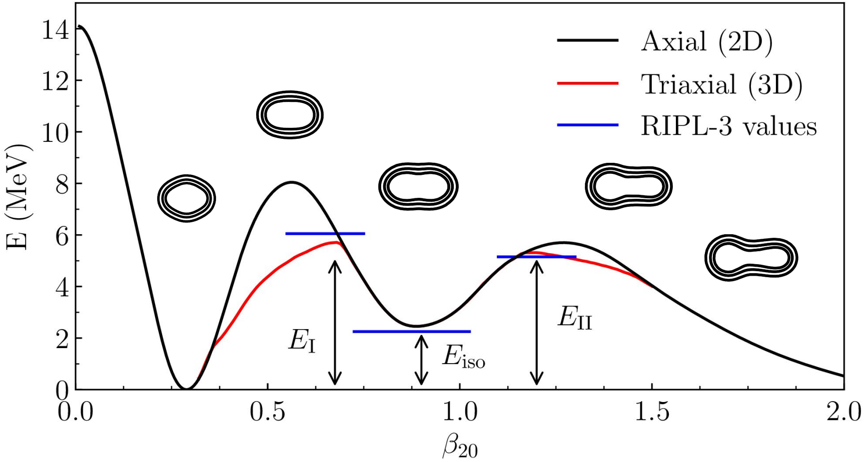


Fission

W. R. et al., EPJA **59**, 96 (2023).
E. Flynn et al., PRC **105** (2022).
S. Bara et al. & A. Sánchez-Fernández et al., in preparation.



Fission properties of 45 actinide nuclei

- includes odd-A and odd-odds
- **__all__** inner barriers: triaxial
- **__all__** outer barriers: octupole + triaxial

... even with consistent inertia's!

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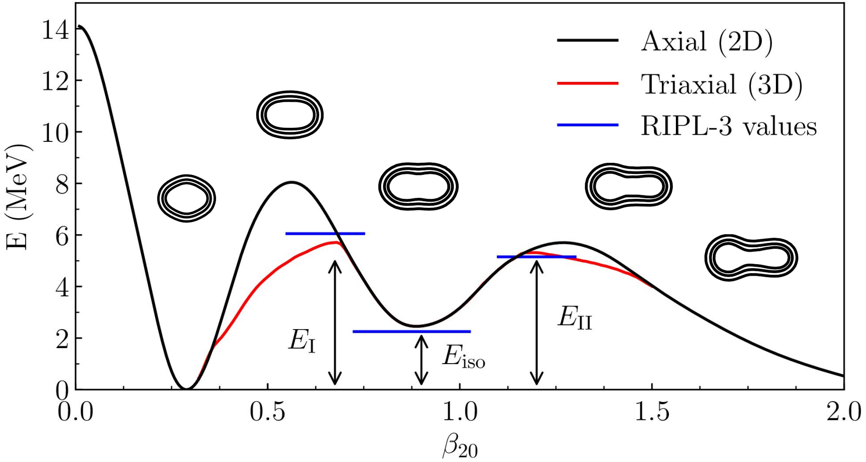

Fission properties of 45 actinide nuclei

- includes odd-A and odd-odds
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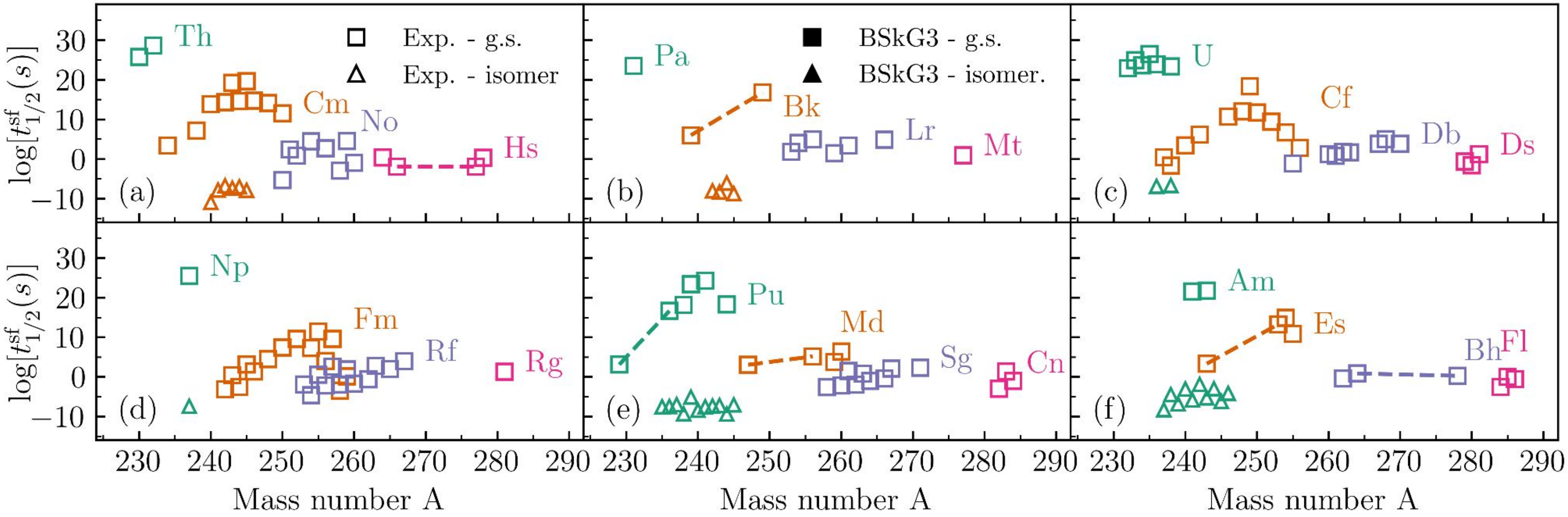
- <u>all</u> outer barriers: octupole + triaxial

… even with consistent inertia's!

Large-scale data generation

- poor-mans 3D coll. space: $\beta_{20,}$, β_{22} , β_{30}
- odd nuclei through EFA
- pathfinding with PyNEB

✅ 1500/3000 nuclei (Z > 78)

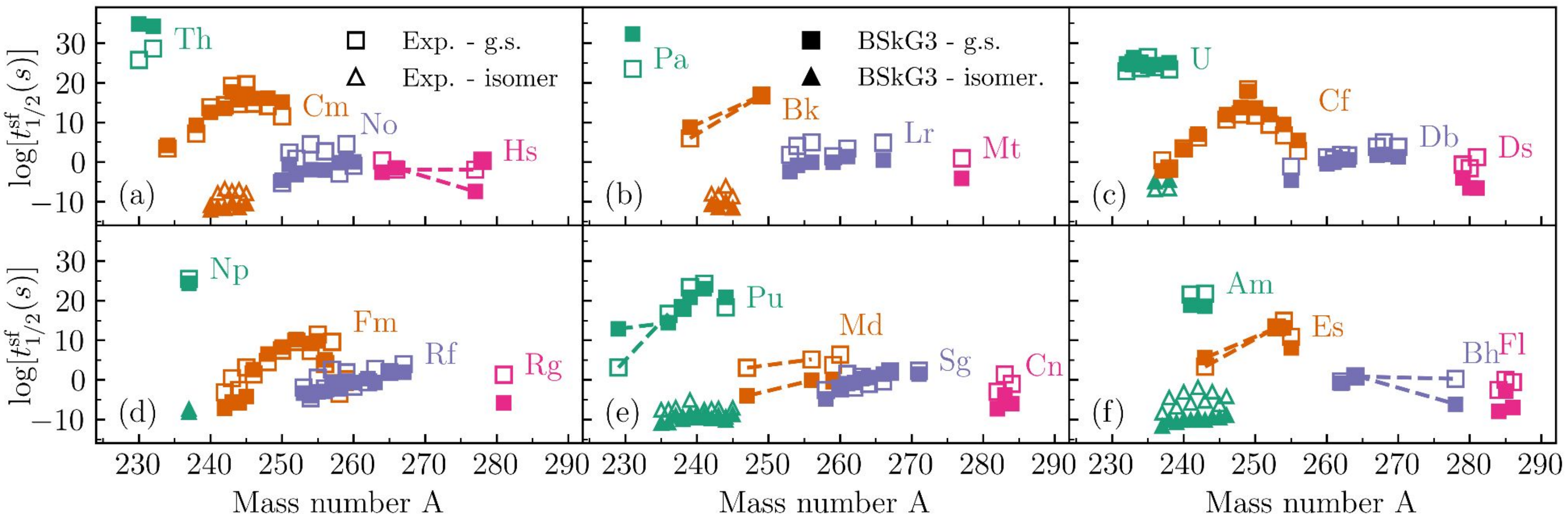


Spontaneous fission half-lives

Experimental data:

- 124 ground state half-lives
- 34 half-lives of isomeric states

... that span **> 40** orders of magnitude!



A. Sánchez-Fernández et al., in prep.

Spontaneous fission half-lives

Experimental data:

- 124 ground state half-lives
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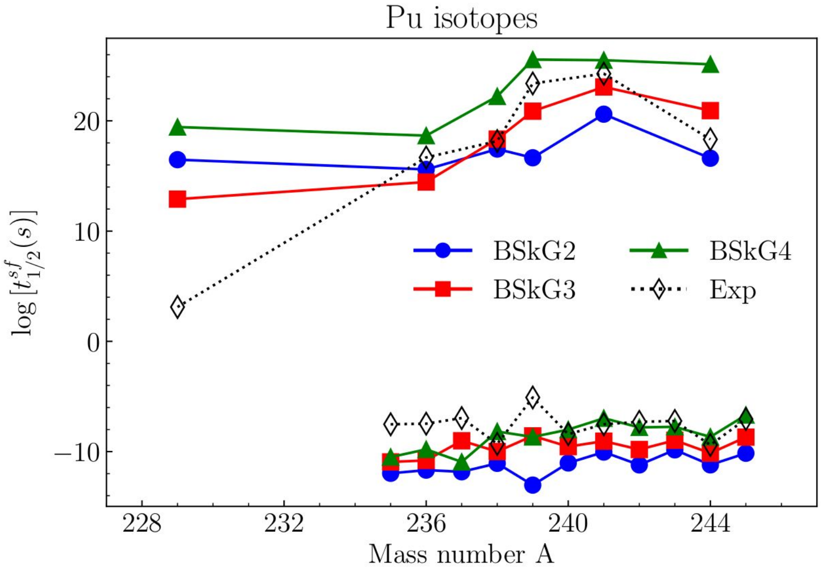
... that span > **40** orders of magnitude!

... at scale from microscopic models!

- comparable best mic-mac models
- ... without adjustable parameters!

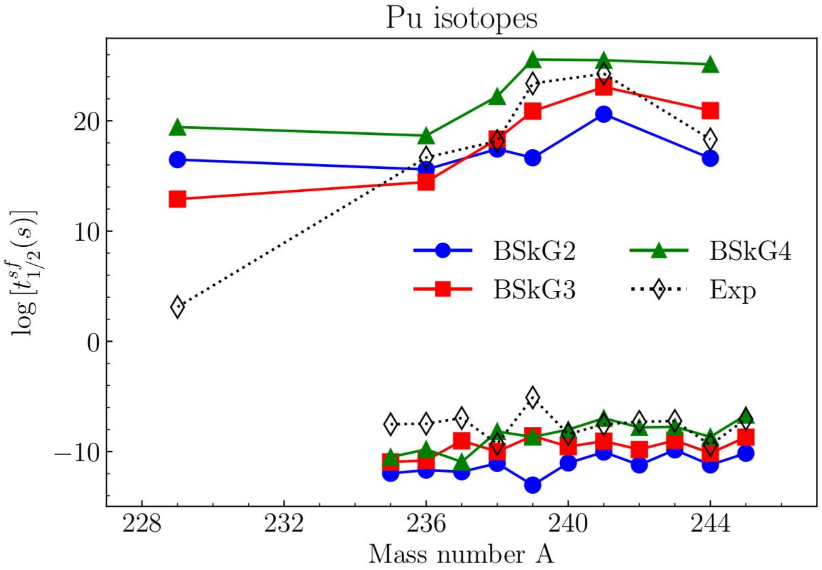
Model dependency and systematics



Model dependency

- BSkG3 is spectacular, BSkG2 /4 decent
- BSkG4 has weaker and more "realistic" pairing.

Model dependency and systematics



$$r_i = \frac{t^{\mathrm{sf}}_{1/2,\mathrm{calc}}}{t^{\mathrm{sf}}_{1/2,\mathrm{exp}}}$$

$$f_{\mathrm{rms}} = \exp\left[\frac{1}{N}\sum_i^N (\ln r_i)^2\right]^{1/2}, \qquad f_{\mathrm{mean}} = \exp\left[\frac{1}{N}\sum_i^N \ln r_i\right].$$

	f_{rms}			f_{mean}		
	BSkG2	BSkG3	BSkG4	BSkG2	BSkG3	BSkG4
all (124)	$1.8{\cdot}10^5$	$3.1{\cdot}10^3$	$3.2{\cdot}10^4$	10^{-3}	$7{\cdot}10^{-2}$	$1.5{\cdot}10^2$
even (59)	$3{\cdot}10^4$	$7.6{\cdot}10^2$	$5.2{\cdot}10^4$	$3{\cdot}10^{-2}$	1.5	$1.2{\cdot}10^3$
odd (65)	$7.6{\cdot}10^5$	$9.2{\cdot}10^3$	$2{\cdot}10^4$	10^{-4}	$4{\cdot}10^{-3}$	$2.4{\cdot}10^1$

Model dependency

- BSkG3 is spectacular, BSkG2 /4 decent
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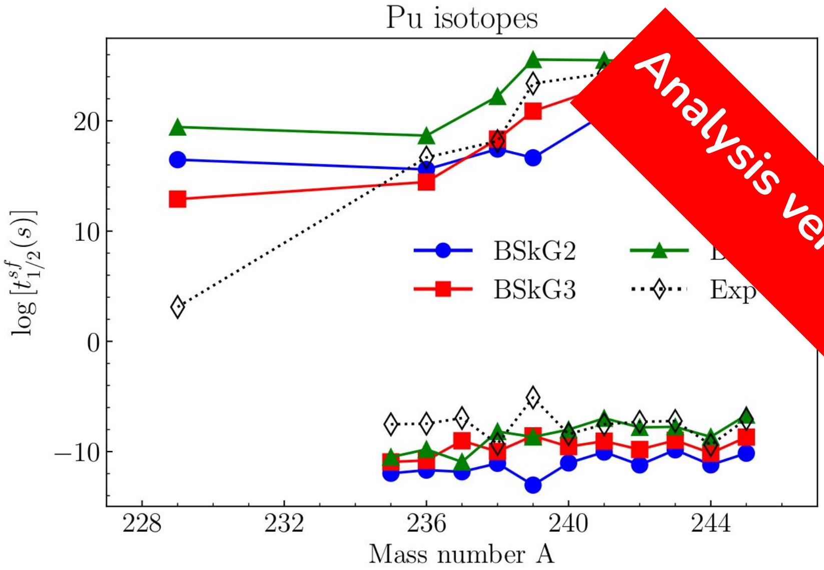
Systematics for BSkG3

- fission is too fast; mostly odds
- short half-lives underestimated
- but only ~45/124 nuclei were in the fit
- ❓ actual observables in the fit

Model dependency and systematics



Analysis very much ongoing!

$$r_i = \frac{t^{\rm sf}_{1/2,\rm calc}}{t^{\rm sf}_{1/2,\rm exp}}$$

$$f_{\rm rms} = \exp\left[\frac{1}{N}\sum_i^N (\ln r_i)^2\right]^{1/2}, \qquad f_{\rm mean} = \exp\left[\frac{1}{N}\sum_i^N \ln r_i\right].$$

	$f_{\rm rms}$ BSkG2	$f_{\rm rms}$ BSkG3	$f_{\rm rms}$ BSkG4	$f_{\rm mean}$ BSkG2	$f_{\rm mean}$ BSkG3	$f_{\rm mean}$ BSkG4
	$1.8{\cdot}10^5$	$3.1{\cdot}10^3$	$3.2{\cdot}10^4$	10^{-3}	$7{\cdot}10^{-2}$	$1.5{\cdot}10^2$
	$3{\cdot}10^4$	$7.6{\cdot}10^2$	$5.2{\cdot}10^4$	$3{\cdot}10^{-2}$	1.5	$1.2{\cdot}10^3$
od	10^5	$9.2{\cdot}10^3$	$2{\cdot}10^4$	10^{-4}	$4{\cdot}10^{-3}$	$2.4{\cdot}10^1$

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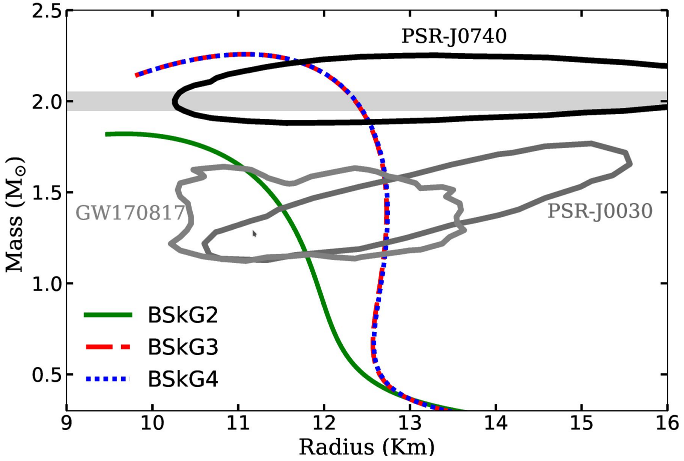
Neutron stars

G. Grams et al., EPJA **59**, 270 (2023).



Stiffer EoS

- results in higher maximum mass
- usually incompatible with masses

Neutron stars

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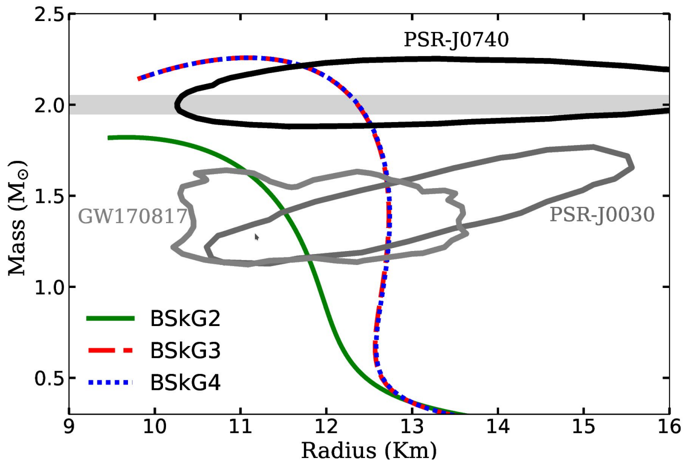

$$
\begin{aligned}
\mathcal{E}_{t,\mathrm{e}}(\mathbf{r}) = {} & C_t^{\rho\rho}(\rho_0)\,\rho_t^2(\mathbf{r}) + \boxed{C_t^{\rho\tau}(\rho_0)}\,\rho_t(\mathbf{r})\,\tau_t(\mathbf{r}) \\
& + C_t^{\rho\nabla J}\,\rho_t(\mathbf{r})\,\nabla\cdot\mathbf{J}_t(\mathbf{r}) \\
& + C_t^{\rho\Delta\rho}\,\rho_t(\mathbf{r})\,\Delta\rho_t(\mathbf{r}) \\
& + \boxed{C_t^{\nabla\rho\nabla\rho}(\rho_0)}\,\nabla\rho_t(\mathbf{r})\cdot\nabla\rho_t(\mathbf{r}) \\
& + \boxed{C_t^{\rho\nabla\rho\nabla\rho}(\rho_0)\,\rho_t(\mathbf{r})\nabla\rho_0(\mathbf{r})\cdot\nabla\rho_t(\mathbf{r})}
\end{aligned}
$$

Stiffer EoS

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Extended density dependencies

- ad-hoc suggestion
- 6 additional parameters

... two are fractional powers!

BSkG5

G. Grams et al., in preparation.

$$
\begin{aligned}
\mathcal{E}^{(0)}_{\mathrm{Sk,e}}(\vec{r}) &= \sum_{t=0,1} \Big[A^{(0,1)}_{\mathrm{t,e}} \left(D^{1,1}_{t} \right)^2 + A^{(0,2)}_{\mathrm{t,e}} \left(D^{1,1}_{0} \right)^{\alpha} \left(D^{1,1}_{t} \right)^2 \Big] , \\
\mathcal{E}^{(2)}_{\mathrm{Sk,e}}(\vec{r}) &= \sum_{t=0,1} \Big[A^{(2,1)}_{\mathrm{t,e}} D^{1,1}_{t} \left(\Delta D^{1,1}_{t} \right) + A^{(2,2)}_{\mathrm{t,e}} D^{1,1}_{t} D^{(\nabla,\nabla)}_{t} + A^{(2,3)}_{\mathrm{t,e}} \sum_{\mu\nu} C^{1,\nabla\sigma}_{t,\mu\nu} C^{1,\nabla\sigma}_{t,\mu\nu} + A^{(2,4)}_{\mathrm{t,e}} D^{1,1}_{t} \left(\nabla \cdot C^{1,\nabla\times\sigma}_{t} \right) \Big] , \\
\mathcal{E}^{(4)}_{\mathrm{Sk,e}}(\vec{r}) &= \sum_{t=0,1} \Big[A^{(4,1)}_{\mathrm{t,e}} \left(\Delta D^{1,1}_{t} \right) \left(\Delta D^{1,1}_{t} \right) + A^{(4,2)}_{\mathrm{t,e}} D^{1,1}_{t} D^{\Delta,\Delta}_{t} + A^{(4,3)}_{\mathrm{t,e}} D^{(\nabla,\nabla)}_{t} D^{(\nabla,\nabla)}_{t} \\
&\quad + A^{(4,4)}_{\mathrm{t,e}} \sum_{\mu\nu} D^{\nabla,\nabla}_{t,\mu\nu} D^{\nabla,\nabla}_{t,\mu\nu} + A^{(4,5)}_{\mathrm{t,e}} \sum_{\mu\nu} D^{\nabla,\nabla}_{t,\mu\nu} \left(\nabla_{\mu} \nabla_{\nu} D^{1,1}_{t} \right) \\
&\quad + A^{(4,6)}_{\mathrm{t,e}} \sum_{\mu\nu} C^{1,\nabla\sigma}_{t,\mu\nu} \left(\Delta C^{1,\nabla\sigma}_{t,\mu\nu} \right) + A^{(4,7)}_{\mathrm{t,e}} \sum_{\mu\nu\kappa} \left(\nabla_{\mu} C^{1,\nabla\sigma}_{t,\mu\kappa} \right) \left(\nabla_{\nu} C^{1,\nabla\sigma}_{t,\nu\kappa} \right) + A^{(4,8)}_{\mathrm{t,e}} \sum_{\mu\nu} C^{1,\nabla\sigma}_{t,\mu\nu} C^{\Delta,\nabla\sigma}_{t,\mu\nu} \Big] , \\
\mathcal{E}^{(0)}_{\mathrm{Sk,o}}(\vec{r}) &= \sum_{t=0,1} \Big[A^{(0,1)}_{\mathrm{t,o}} \vec{D}^{1,\sigma}_{t} \cdot \vec{D}^{1,\sigma}_{t} + A^{(0,2)}_{\mathrm{t,o}} (D^{1,1}_{0})^{\alpha} \vec{D}^{1,\sigma}_{t} \cdot \vec{D}^{1,\sigma}_{t} \Big] , \\
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&\quad + A^{(4,6)}_{\mathrm{t,o}} \vec{C}^{1,\nabla}_{t} \cdot \left(\Delta \vec{C}^{1,\nabla}_{t} \right) + A^{(4,7)}_{\mathrm{t,o}} \left(\boldsymbol{\nabla} \cdot \vec{C}^{1,\nabla}_{t} \right) \left(\boldsymbol{\nabla} \cdot \vec{C}^{1,\nabla}_{t} \right) + A^{(4,8)}_{\mathrm{t,o}} \vec{C}^{1,\nabla}_{t} \cdot \vec{C}^{\Delta,\nabla}_{t} \Big] ,
\end{aligned}
$$

N2LO EDF

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BSkG5

G. Grams et al., in preparation.

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&\quad + A^{(4,6)}_{\mathrm{t,e}} \sum_{\mu\nu} C^{1,\nabla\sigma}_{t,\mu\nu} \left(\Delta C^{1,\nabla\sigma}_{t,\mu\nu}\right) + A^{(4,7)}_{\mathrm{t,e}} \sum_{\mu\nu\kappa} \left(\nabla_\mu C^{1,\nabla\sigma}_{t,\mu\kappa}\right) \left(\nabla_\nu C^{1,\nabla\sigma}_{t,\nu\kappa}\right) + A^{(4,8)}_{\mathrm{t,e}} \sum_{\mu\nu} C^{1,\nabla\sigma}_{t,\mu\nu} C^{\Delta,\nabla\sigma}_{t,\mu\nu} \Big], \\
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&\quad + A^{(4,6)}_{\mathrm{t,o}} \vec{C}^{1,\nabla}_t \cdot \left(\Delta \vec{C}^{1,\nabla}_t\right) + A^{(4,7)}_{\mathrm{t,o}} \left(\boldsymbol{\nabla} \cdot \vec{C}^{1,\nabla}_t\right) \left(\boldsymbol{\nabla} \cdot \vec{C}^{1,\nabla}_t\right) + A^{(4,8)}_{\mathrm{t,o}} \vec{C}^{1,\nabla}_t \cdot \vec{C}^{\Delta,\nabla}_t \Big],
\end{aligned}
$$

N2LO EDF

- **systematic** expansion in gradients
- next-to-next-to-leading order
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- massively complicates things

… what if we improve INM first?

BSkG5

G. Grams et al., in preparation.

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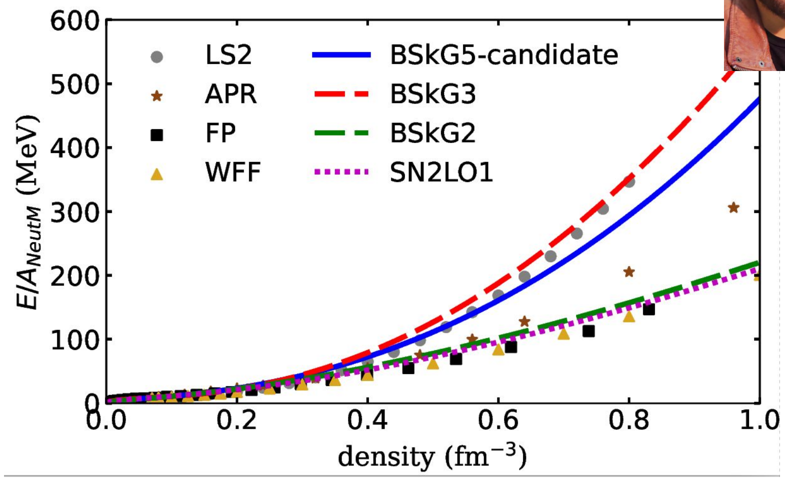
\end{aligned}
$$



N2LO EDF

- **systematic** expansion in gradients
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… what if we improve INM first?

- same global quality as BSkG4
- density dependence as standard Skyrme

BSkG5

G. Grams et al., in preparation.

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&\quad + A^{(4,4)}_{\mathrm{t,e}} \sum_{\mu\nu} D^{\nabla,\nabla}_{t,\mu\nu} D^{\nabla,\nabla}_{t,\mu\nu} + A^{(4,5)}_{\mathrm{t,e}} \sum_{\mu\nu} D^{\nabla,\nabla}_{t,\mu\nu} \left(\nabla_\mu \nabla_\nu D^{1,1}_t\right) \\
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\mathcal{E}^{(0)}_{\mathrm{Sk,o}}(\vec{r}) &= \sum_{t=0,1} \left[A^{(0,1)}_{\mathrm{t,o}} \vec{D}^{1,\sigma}_t \cdot \vec{D}^{1,\sigma}_t + A^{(0,2)}_{\mathrm{t,o}} (D^{1,1}_0)^{\alpha} \vec{D}^{1,\sigma}_t \cdot \vec{D}^{1,\sigma}_t \right], \\
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\end{aligned}
$$

Rms σ	BSkG4	BSkG5
Masses [MeV]	0.633	0.637
S_n [MeV]	0.402	0.407
Radii [fm]	0.025	0.028
Prim. barriers [MeV]	0.36	0.36
Sec. barriers [MeV]	0.53	0.53
Fission isomers [MeV]	0.33	0.42

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$$
\begin{aligned}
\mathcal{E}^{(0)}_{\mathrm{Sk,e}}(\vec{r}) &= \sum_{t=0,1}\Big[A^{(0,1)}_{\mathrm{t,e}}\left(D^{1,1}_t\right)^2 + A^{(0,2)}_{\mathrm{t,e}}\left(D^{1,1}_0\right)^\alpha\left(D^{1,1}_t\right)^2\Big], \\
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&\quad + A^{(4,4)}_{\mathrm{t,e}}\sum_{\mu\nu}D^{\nabla,\nabla}_{t,\mu\nu}D^{\nabla,\nabla}_{t,\mu\nu} + A^{(4,5)}_{\mathrm{t,e}}\sum_{\mu\nu}D^{\nabla,\nabla}_{t,\mu\nu}\left(\nabla_\mu\nabla_\nu D^{1,1}_t\right) \\
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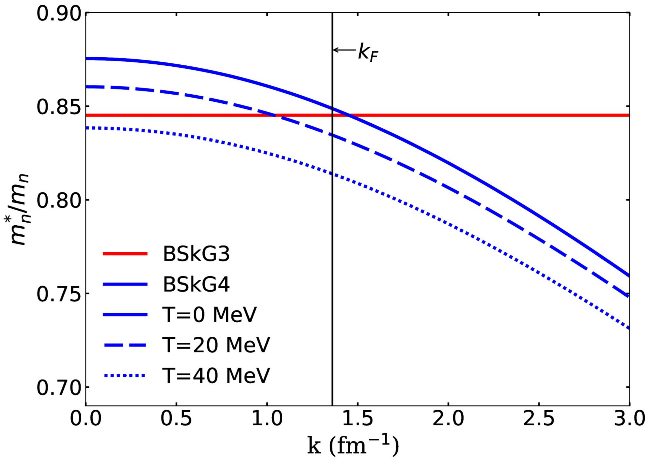

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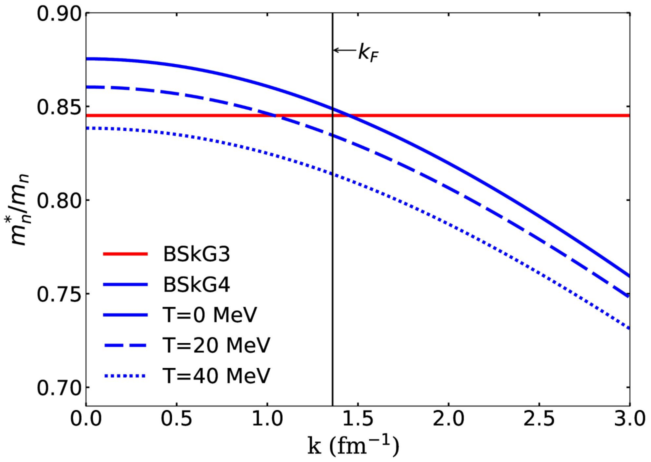

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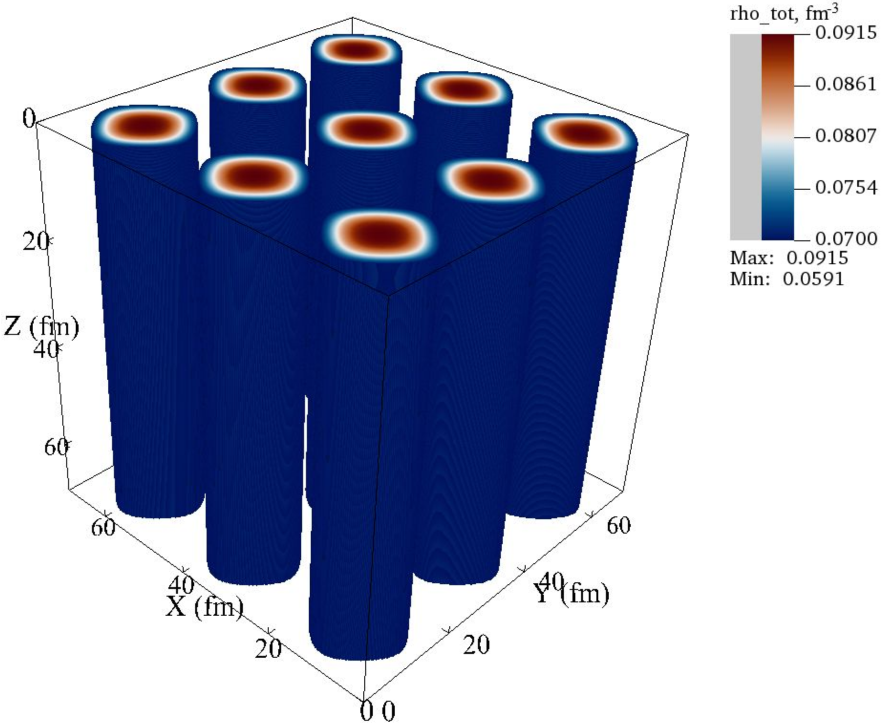
… all with **LESS** parameters!

A complete EoS with the BSkGs

N. Shchechilin et al., in preparation

Nuclear pasta

- crystalline structures in NS crust
- impact NS cooling and emitted GW
- QM treatment with NS-appropriate EDFs
- 10^4 - 10^6 particles in large volumes!



Conclusion

We build **large-scale**, **microscopic** models for (astro) applications.

Large-scale = **thousands** of nuclei and **many observables.**
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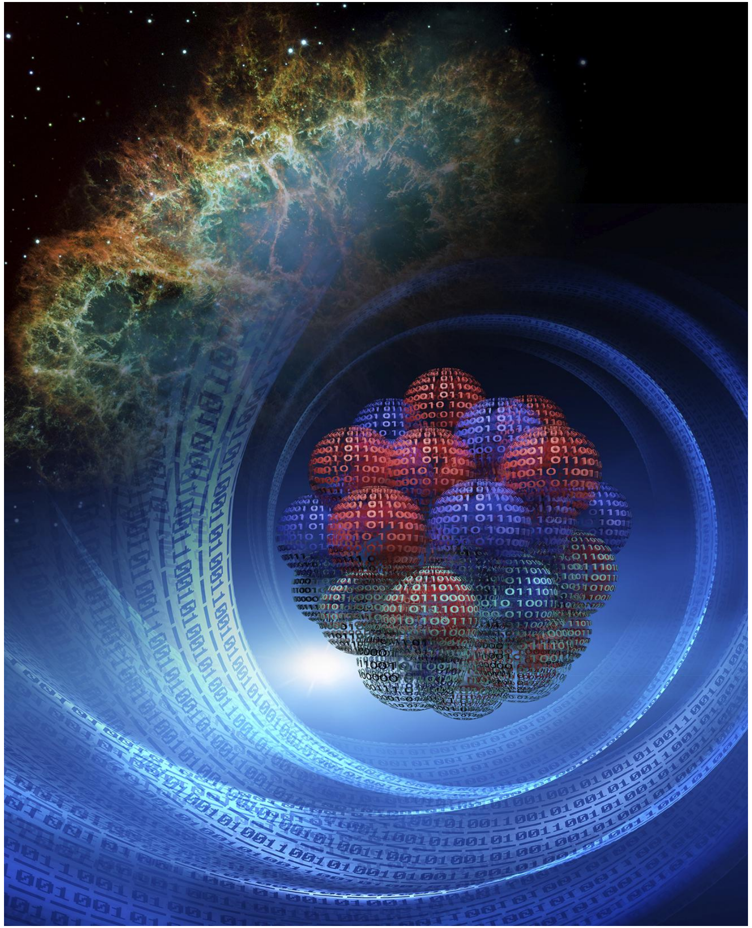

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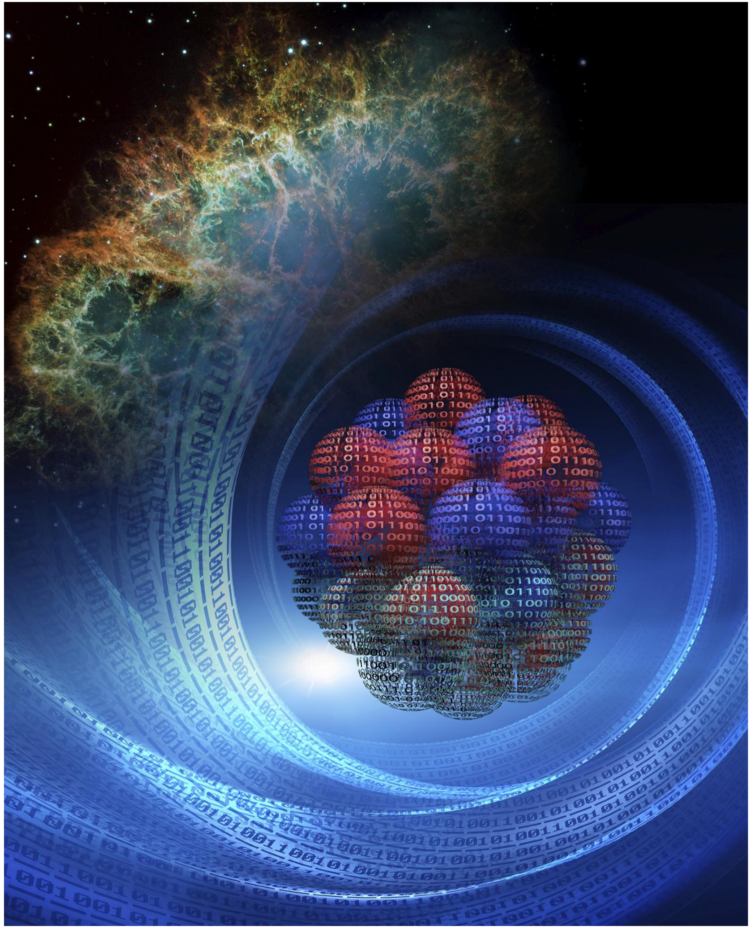
- consistent with NS observations
- masses, densities in TALYS; NLDs soon-to-be



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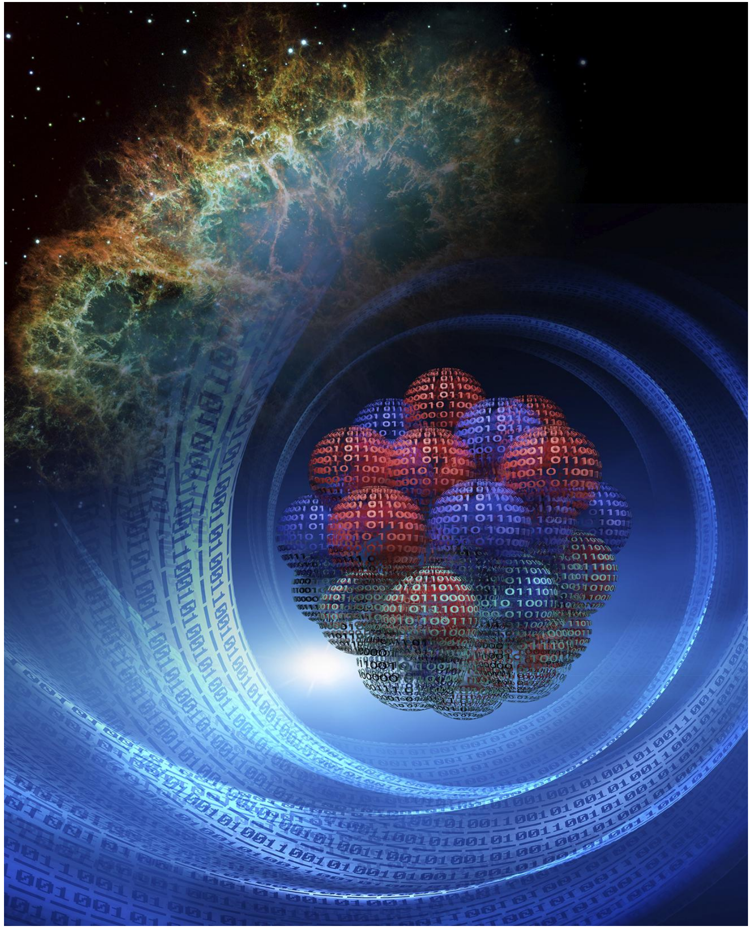

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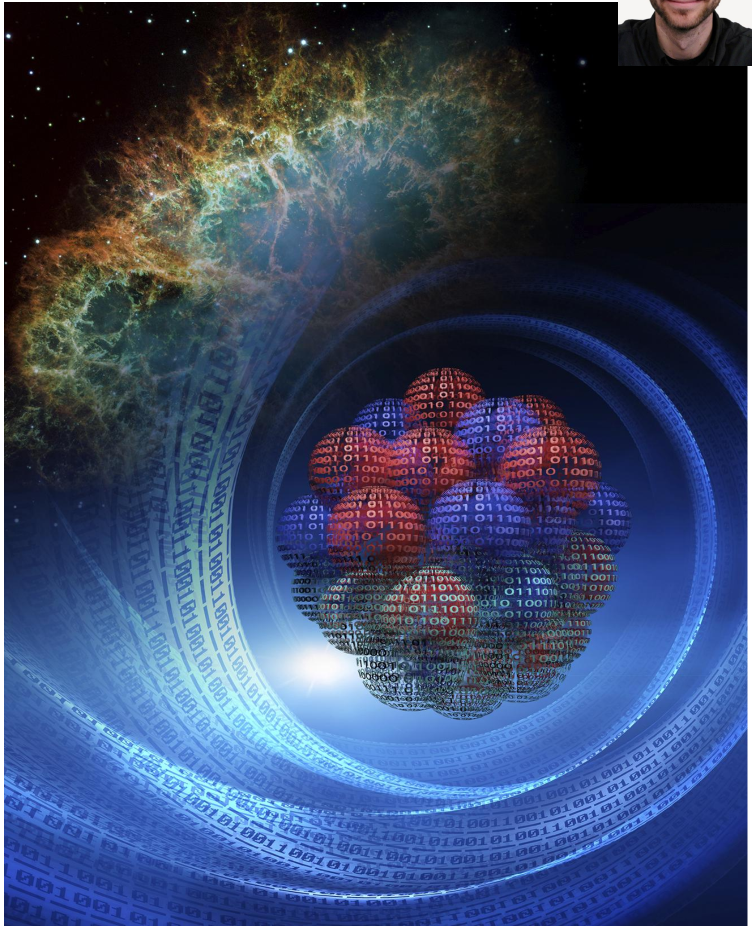

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Thank you for…

…… all the wonderful work!

ULB

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cea

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iP2i LES 2 INFINIS LYON

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CÉCI

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EOS – THE EXCELLENCE OF SCIENCE

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