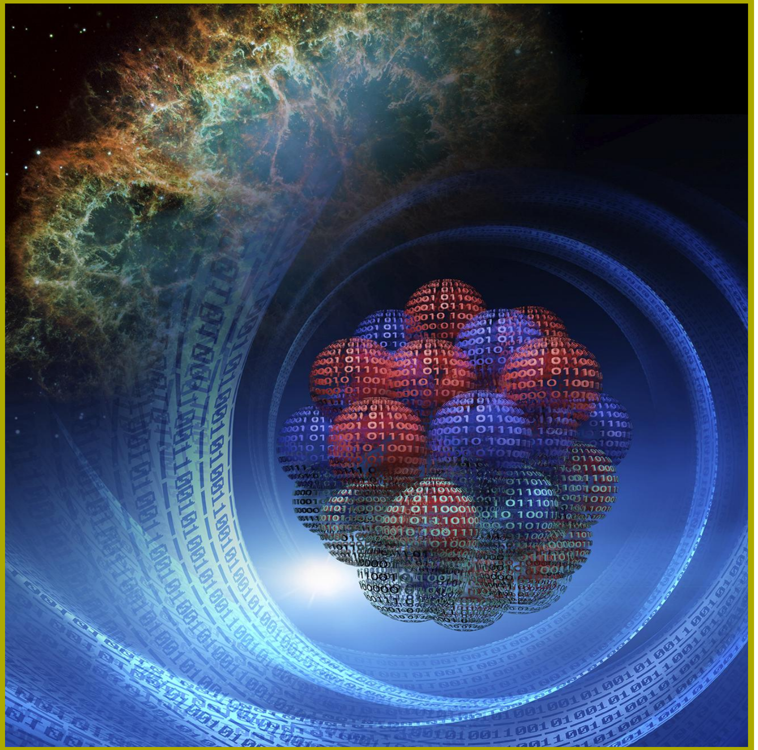
WOUTER RYSSENS



Andy Sproles, ORNL

Nuclear level densities from 3D Brussels Skyrme Models

W. Ryssens, S. Hilaire and S. Goriely

9th of May 2022

wryssens.com

wryssens@ulb.be

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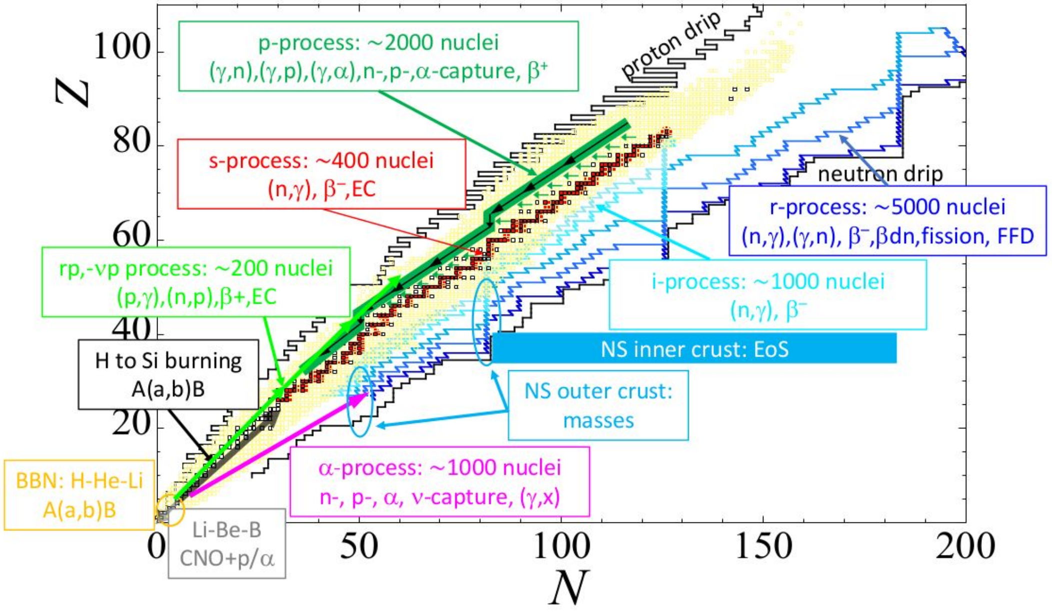
The challenge for nuclear theory



M. Arnould & S. Goriely, Prog. Part. Nuc. Phys. **112,** 103766 (2020).

Skyrme **E**nergy **D**ensity **F**unctionals (**EDF**s)

$$E \sim \int d^3r \left[C^\rho \rho(\mathbf{r})\rho(\mathbf{r}) + C^\tau \tau(\mathbf{r})\rho(\mathbf{r}) + \ldots \right]$$

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Local densities of a mean-field state

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Coupling constants

Local densities of a mean-field state

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Energy (to be varied)

Local densities of a mean-field state

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Energy (to be varied)

Local densities of a mean-field state

Strong points

- **Microscopic** approach
- # of **observables** accessible
- Feasible for **1000's** of nuclei

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Coupling constants

Energy (to be varied)

Local densities of a mean-field state

Strong points

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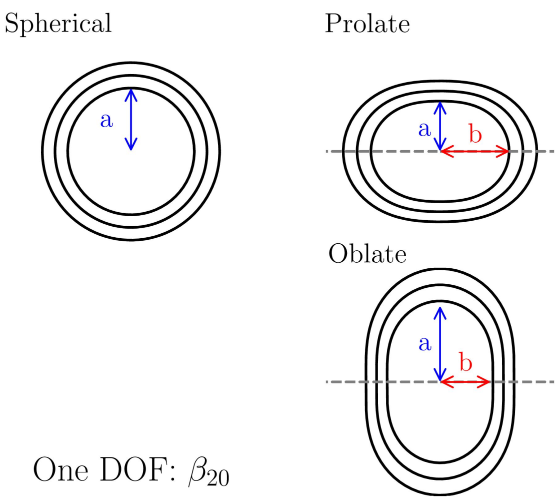
Brussels models

- **20-25** parameters
- grasp physics of the entire chart
- Fit protocol includes:
  - 1000's of masses
  - 100's of charge radii
  - INM properties

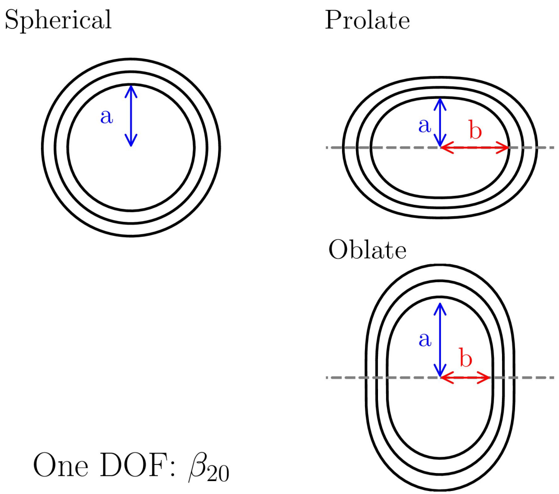
Symmetry breaking



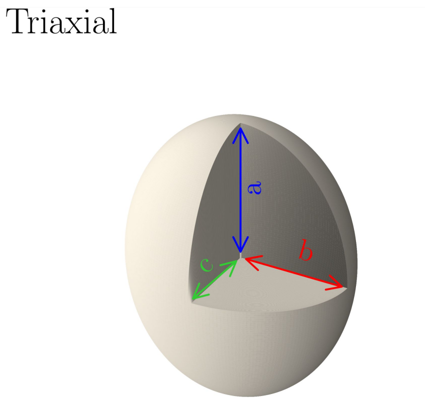
Past models

- time-reversal invariance
- axial symmetry
- reflection symmetry

Symmetry breaking



One DOF: β_{20}



Two DOF: (β_{20}, β_{22}) or (β, γ)

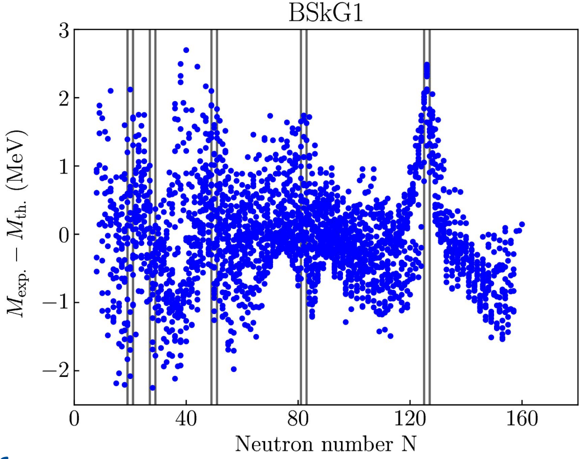
Past models

- time-reversal invariance
- axial symmetry
- reflection symmetry

Todays models

- time-reversal breaking
- triaxial shapes
- reflection symmetric

Brussels Skyrme on a Grid: **BSkG1/2**

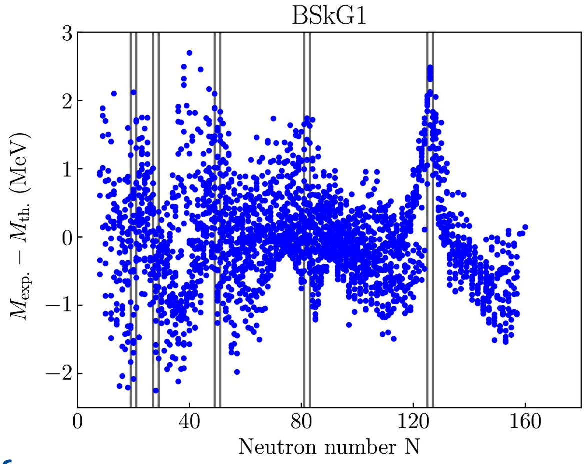


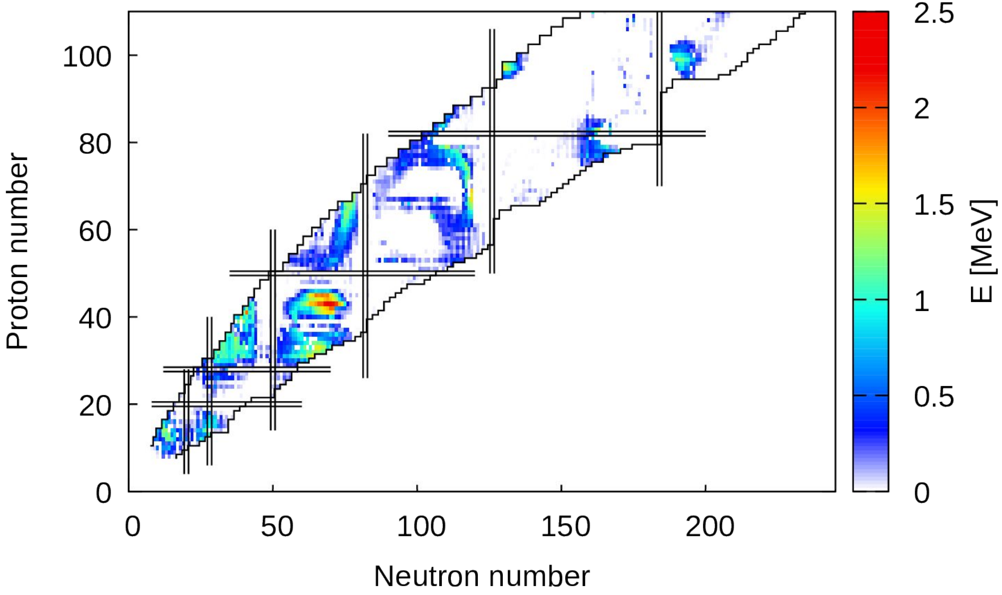
Global performance

σ^2(MeV)	**BSkG1**	**BSkG2**	HFB-14	FR(L)DM
Masses	0.741	**0.678**	0.729	0.808
Radii (fm)	0.027	**0.027**	0.039	-

BSkG1: G. Scamps et al., EPJA 57, 333 (2021);
HFB-14: S. Goriely et al., PRC ***75***, 064312 (2007).
FRDM: P. Möller et al., At. Data Nucl. Data Tables, **109-110** (2016).

Brussels Skyrme on a Grid: **BSkG1/2**





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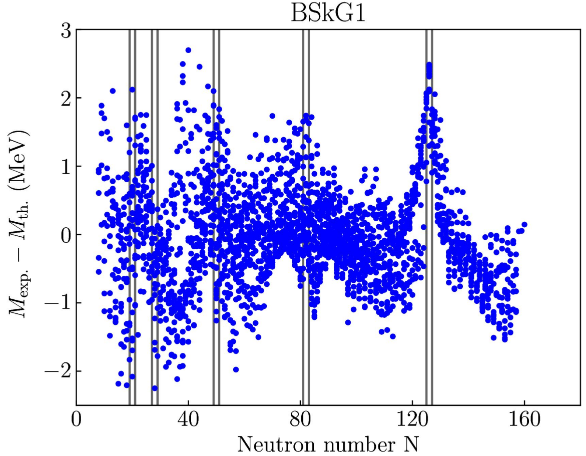

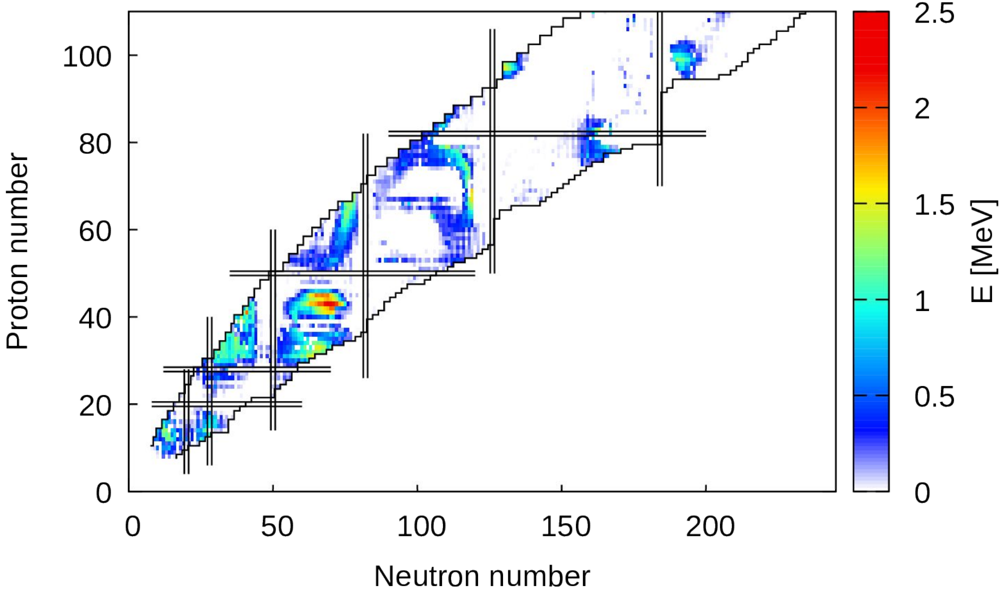

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Masses	0.741	**0.678**	0.729	0.808
Radii (fm)	0.027	**0.027**	0.039	-
Primary barriers	0.87	**0.45**	0.61	0.79
Secondary barriers	0.86	**0.45**	0.70	1.35
Isomers	0.45	**0.48**	0.93	1.04

BSkG1: G. Scamps et al., EPJA 57, 333 (2021);
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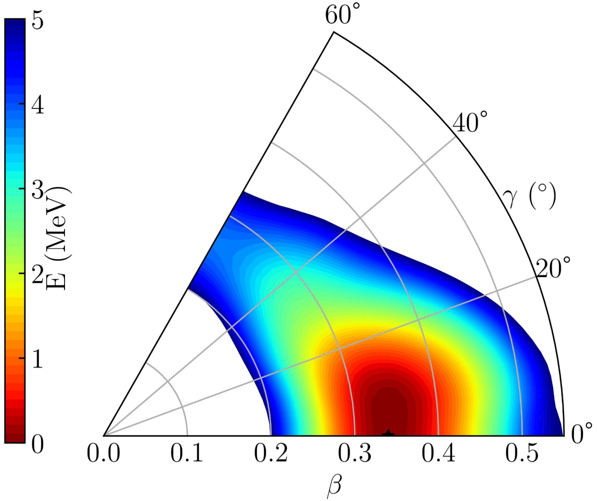
NLDs for axial nuclei: ^{162}Dy

Comb. approach: S. Hilaire, PhD Thesis (1997).
Skyrme: S. Goriely, S. Hilaire and A. J. Koning, PRC 78, 064307 (2008).
Exp. data : NNDC and S. Raman et al. *At. Data Nucl. Data Tables*, *78* (2001).



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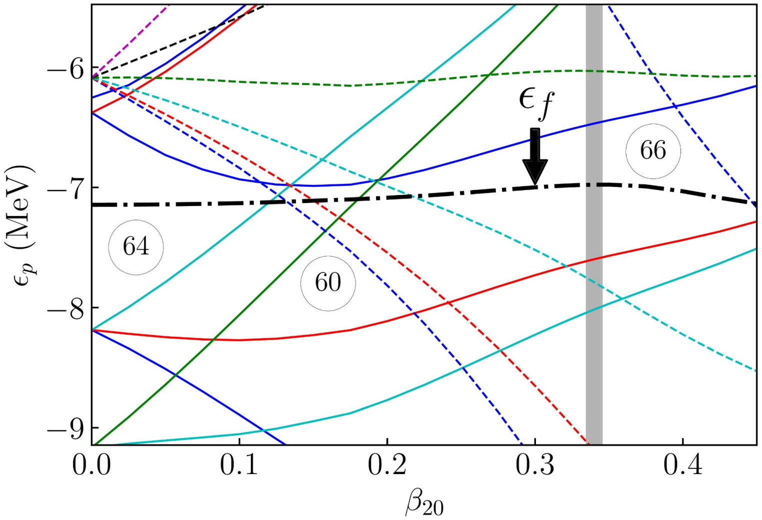

Single-particle aspects

- s.p. level schemes from calculations
- s.p. **K** and **P** quantum numbers
- p-h excitations build intrinsic states
- deformation **<u>decreases</u>** intrinsic NLD

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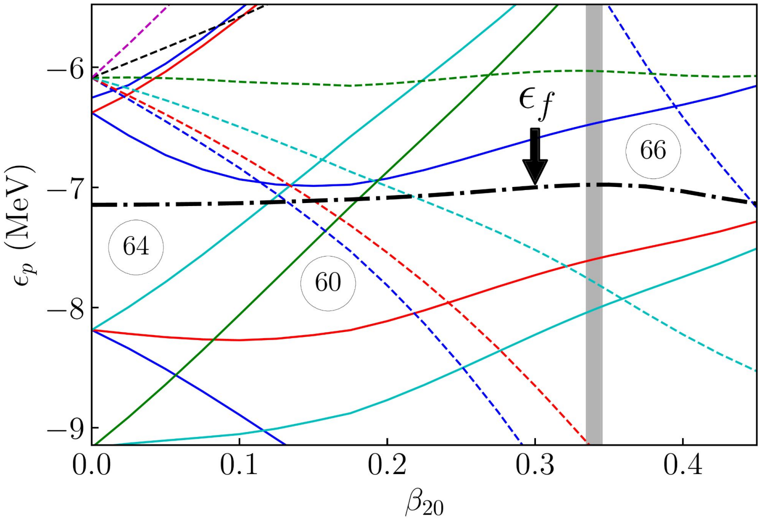
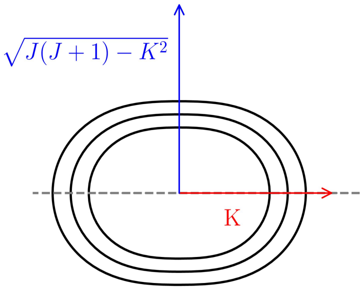
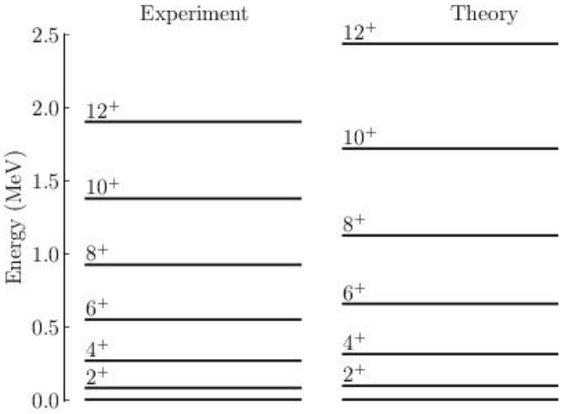






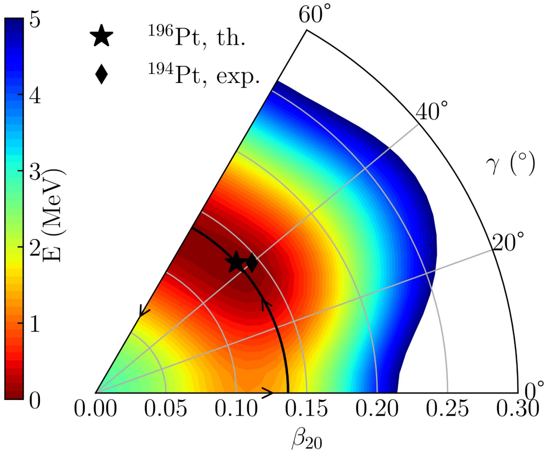
Single-particle aspects

- s.p. level schemes from calculations
- s.p. **K** and **P** quantum numbers
- p-h excitations build intrinsic states
- deformation **decreases** intrinsic NLD

Rotational enhancement

- moments of inertia from calculations
- collective rotation around two axes
- intrinsic K around one axis
- collective enhancement **increases** NLD

NLDs for triaxial nuclei: ^{196}Pt



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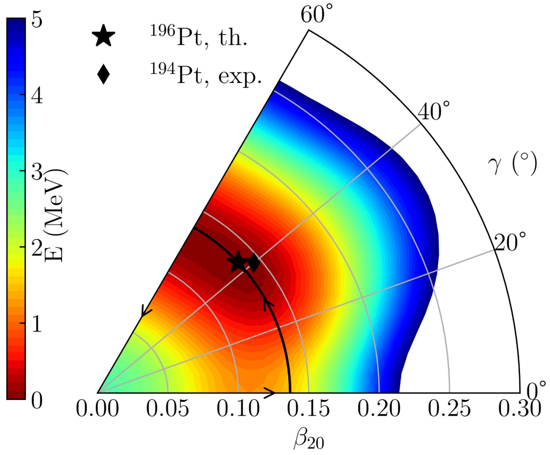
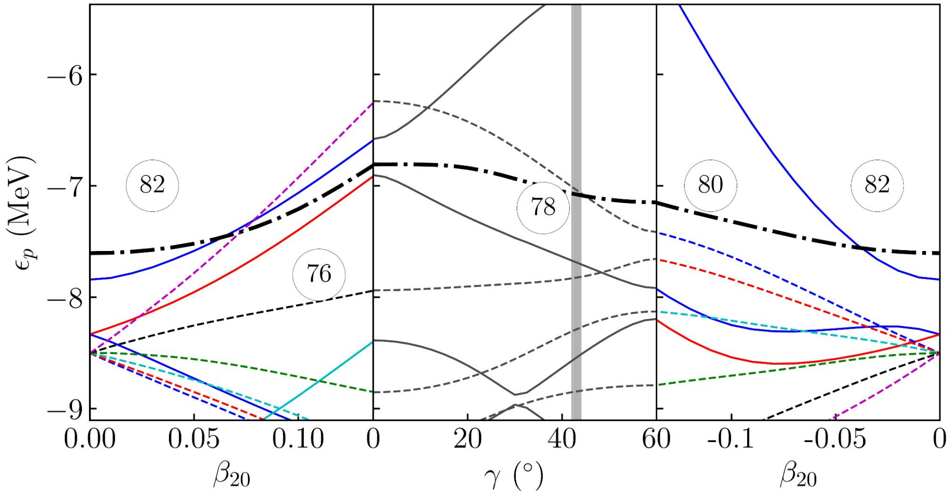
NLDs for triaxial nuclei: ^{196}Pt





Single-particle aspects

- **even lower** s.p. density
- **lower intrinsic** level density
- **no more K** quantum numbers

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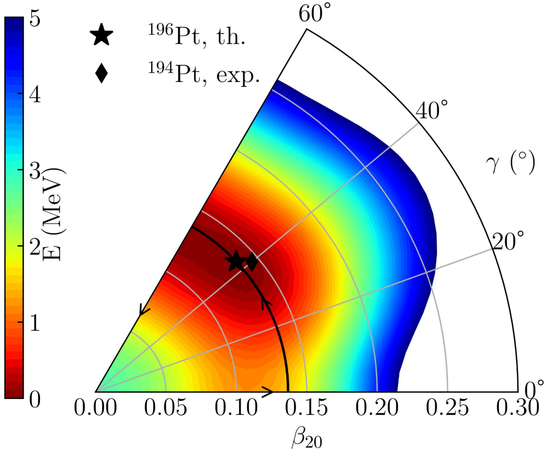
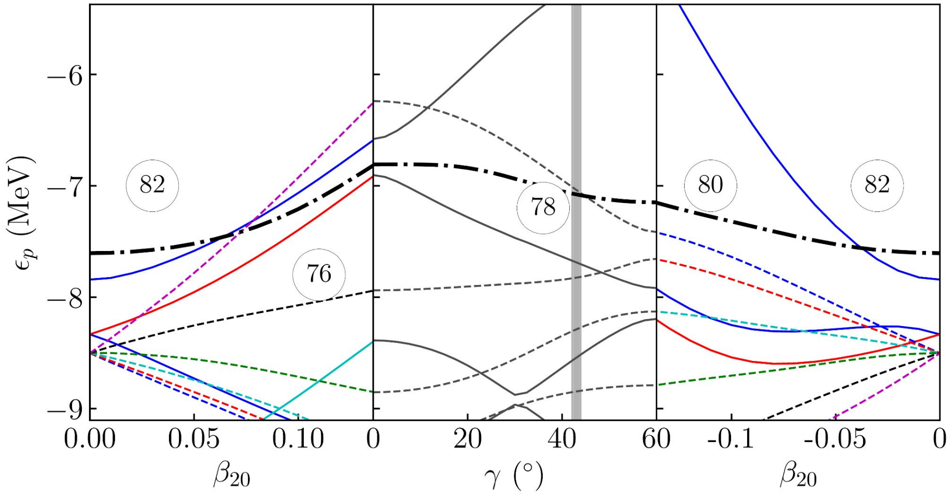
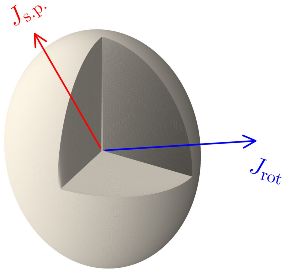
NLDs for triaxial nuclei: ^{196}Pt





Single-particle aspects

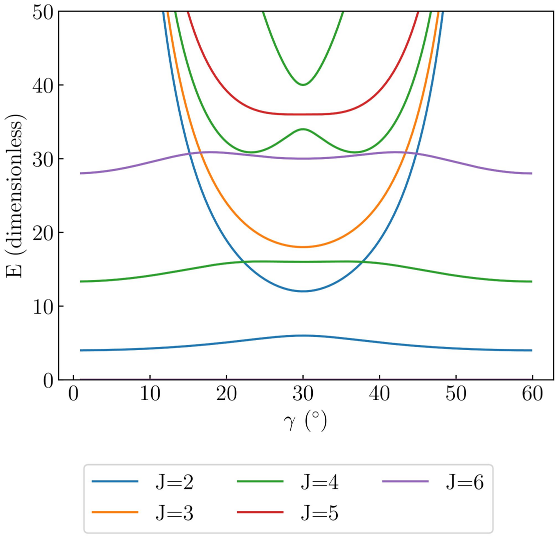
- **even lower** s.p. density
- **lower intrinsic** level density
- **no more K** quantum numbers
- **naive** J-coupling



$$J = \lfloor J_z \rceil + J_{\text{rot}} \, .$$

Exp. data (194Pt): C.Y. Wu et al., Nucl. Phys. **A607**, 178 (1996)

NLDs for triaxial nuclei: ^{196}Pt



Rotational enhancement

- rigid rotor modelling
  - **three** moments of inertia
  - requires a small diagonalization

Exp.: NNDC.

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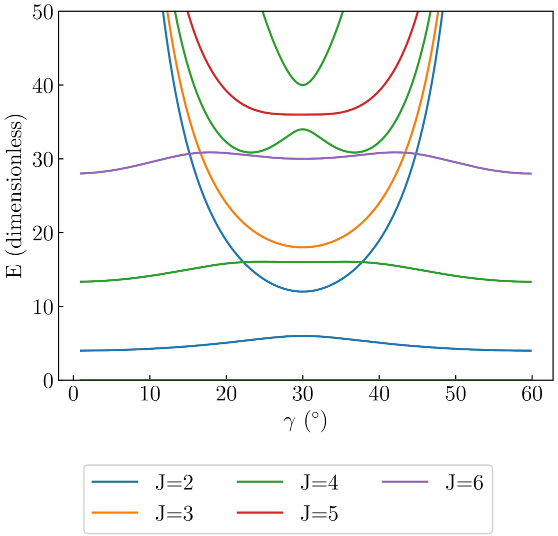
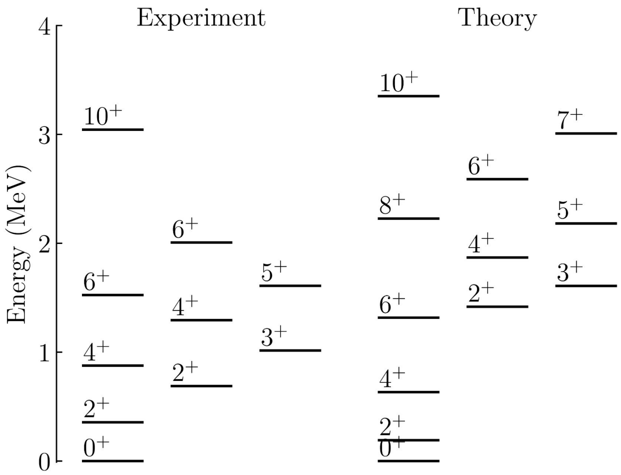


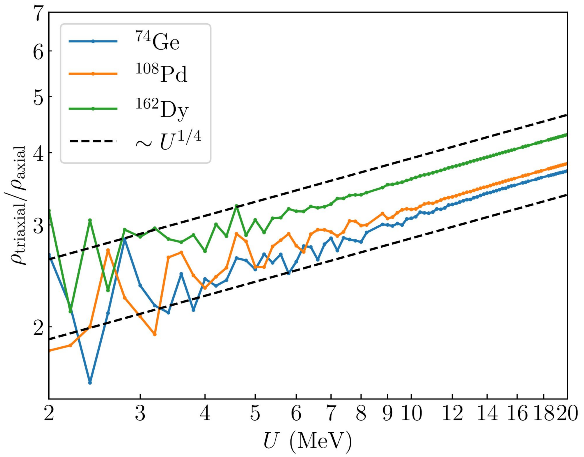


Rotational enhancement

- rigid rotor modelling
  - **three** moments of inertia
  - requires a small diagonalization
- results in (at same excitation)
  - **more states**
  - **more extended spin distribution**

Exp.: NNDC.

Collective enhancement for triaxial nuclei



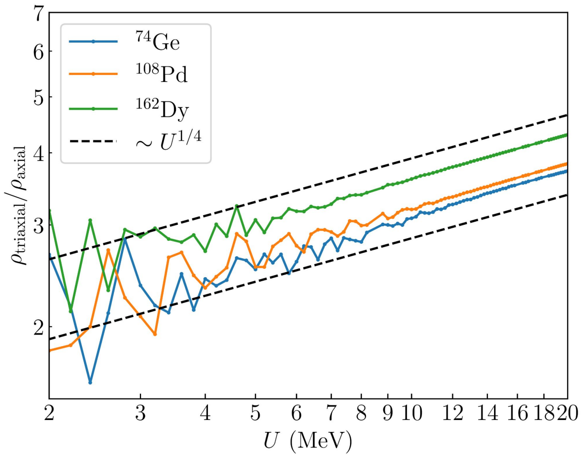
Expectations

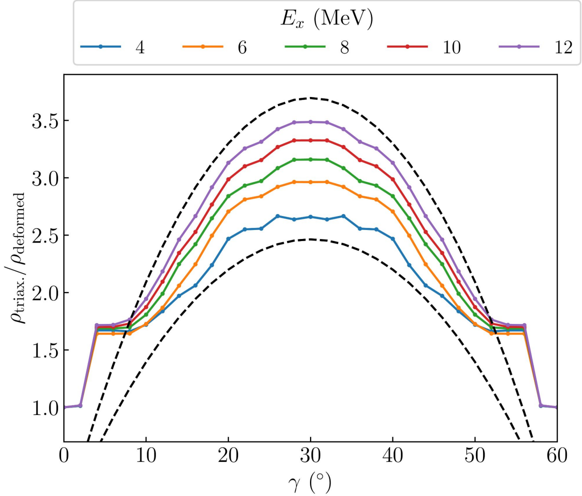
- from analytical models:

$$\frac{\rho_{\rm triaxial}}{\rho_{\rm axial}} \sim \frac{\sqrt{\mathcal{I}_x \mathcal{I}_y \mathcal{I}_z}}{\mathcal{I}_\perp} U^{1/4}$$

- wider spin distributions

Collective enhancement for triaxial nuclei





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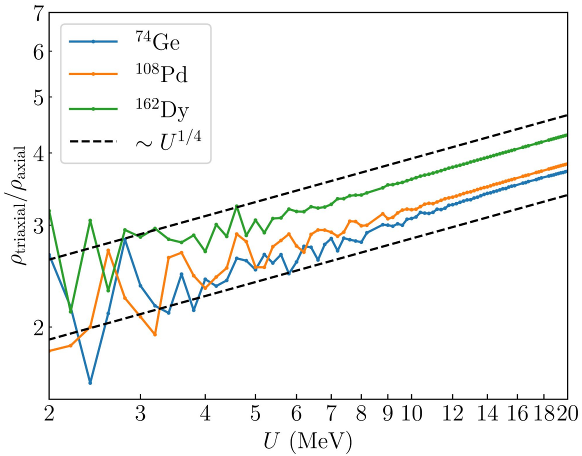

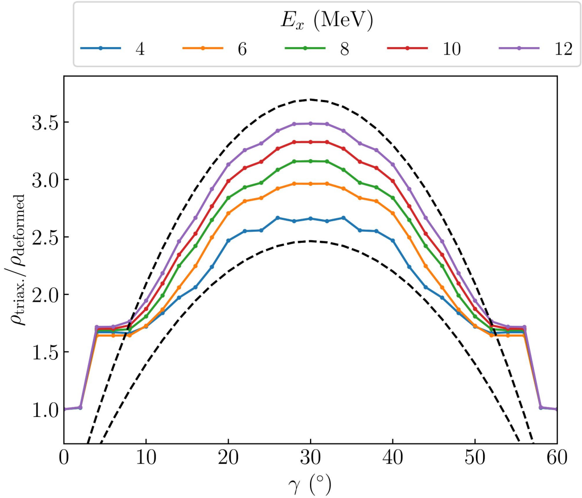

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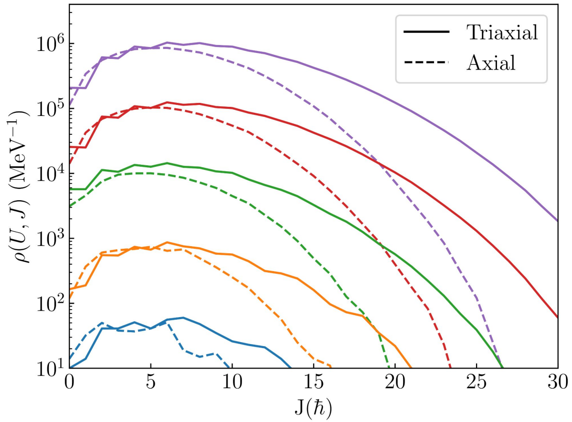
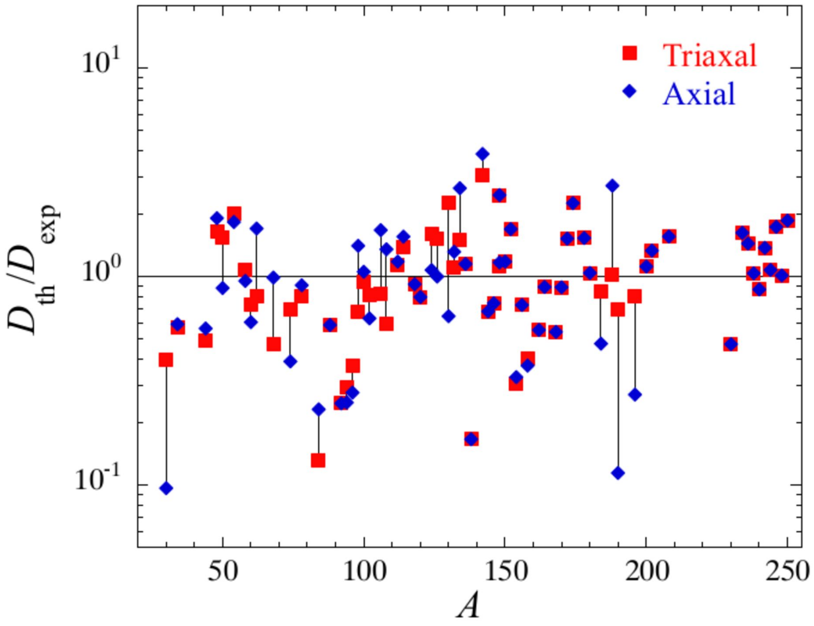


Performance on mean s-wave spacings

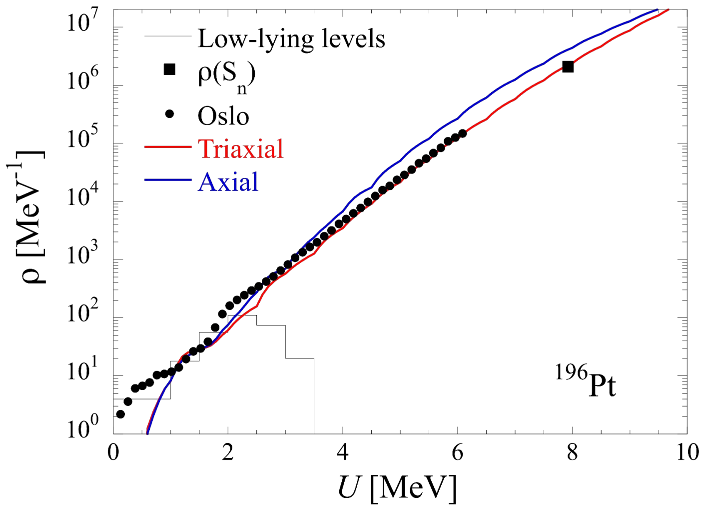
$$f_{\rm rms} = \exp\left[\frac{1}{N_e}\sum_{i=1}^{N_e} \ln^2 \frac{D^i_{\rm th}}{D^i_{\rm exp}}\right]^{1/2},$$

	f_{rms}
BSkG2 (triaxial)	**1.83**
BSkG2 (axial)	**2.13**
BSFG	1.80
HFB+comb	2.30
THFB+comb	2.70



HFB+comb: S. Goriely, S. Hilaire and A. J. Koning, PRC 78, 064307 (2008).
THFB+comb: S. Hilaire, M. Girod, S. Goriely and A. J. Koning, PRC 86, 064317 (2012).
BSFG: A.J. Koning, S. Hilaire and S. Goriely, NPA810, 13-76 (2008).
Exp. data : NNDC and F. Giacoppo, PRC ***90***, 054330 (2014).

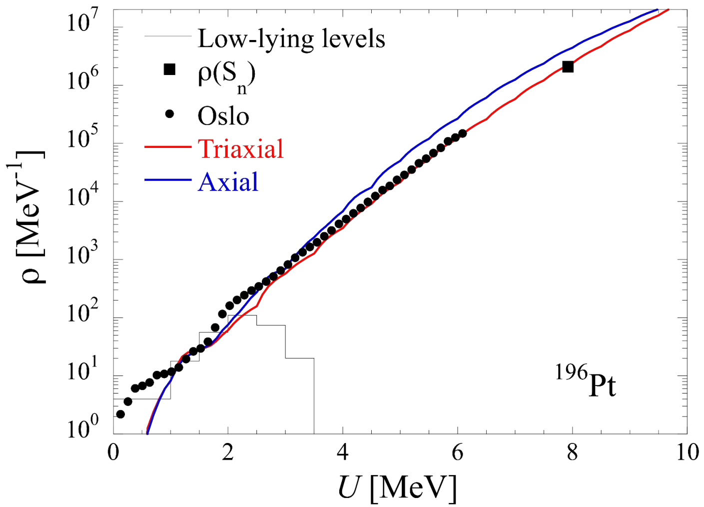
The effect of triaxiality

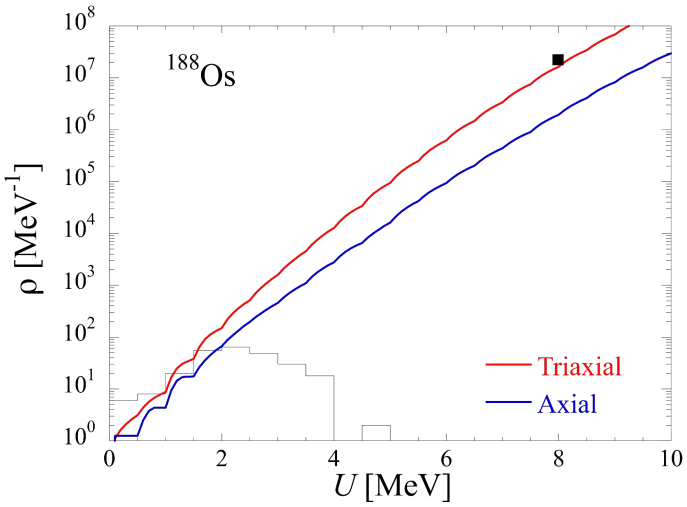


- lower intrinsic NLD
- modest deformation and MOI

Lower overall level density
with a **different U-dependence**

The effect of triaxiality





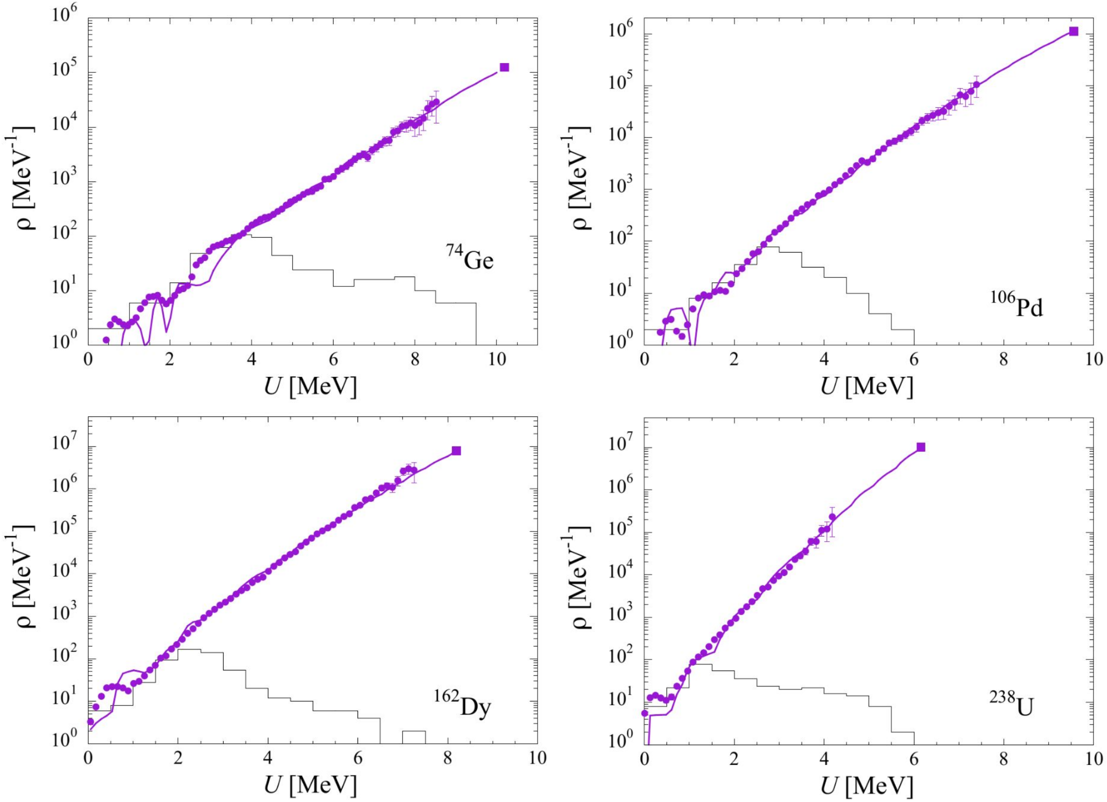
- lower intrinsic NLD
- modest deformation and MOI

Lower overall level density
with a different **U-dependence**

- lower intrinsic NLD
- large deformation and MOI

Larger overall level density
with a different **U-dependence**

Comparing to Oslo data



74Ge: T. Renstrøm, PRC **93**, 064302 (2010). **162Dy :** **M.** Guttormsen et al., PRC ***68***, 064306 (2003).
106Pd: T. Eriksen, PRC **90**, 044311 (2014). **238U:** M. Guttormsen, PRC **89**, 014302 (2014).

Conclusions and outlook

The BSkG2 model:

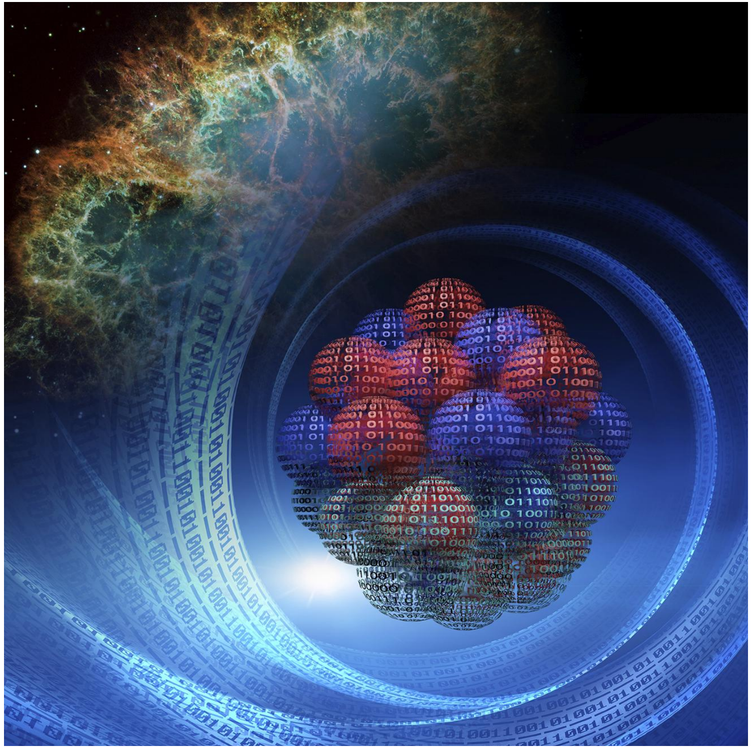
- excellent for all known masses and radii
- includes triaxiality and time-odd channel
- state-of-the-art description of fission barriers

Level densities with BSkG2:

- triaxiality implies
  - **lower s.p. density near Fermi energy**
  - **larger rotational enhancement**
  - **more extended spin distribution**
- many questions remain
  - vibrations?
  - fade-out of triaxiality?
  - large-scale comparisons?

The future:

- complete fission model
- octupole deformation
- neutron-matter equation of state



Thank you for...

...... the help!

ULB

S. Goriely
G. Grams

cea

S. Hilaire

W UNIVERSITY of WASHINGTON

G. Scamps

iP2i LES 2 INFINIS LYON

M. Bender

...... the computing time!

CÉCI

Cenaero

...... the funding!

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...... your attention!