

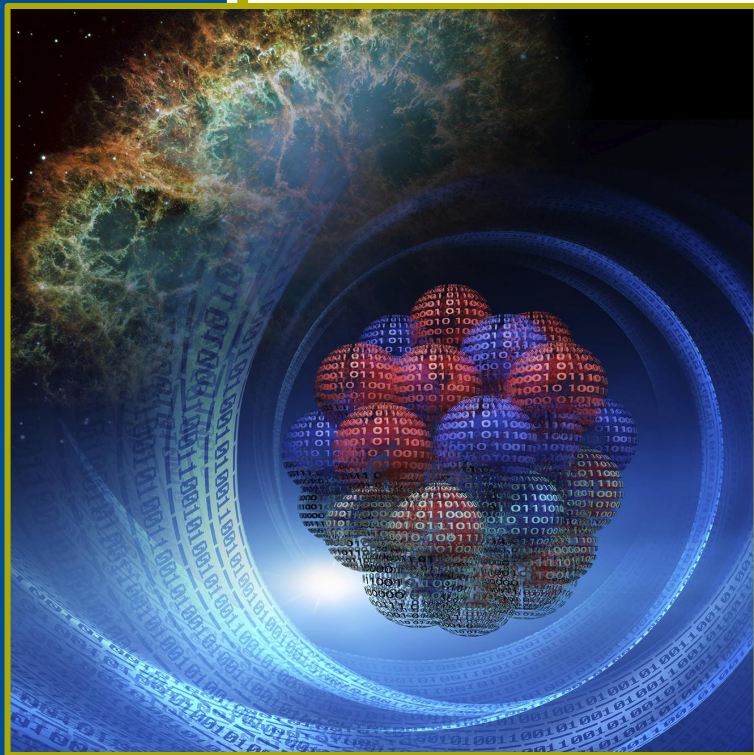


Image credit: Andy Sproles, ORNL

Nuclear mass models for astrophysics applications

W. Ryssens, G. Scamps, G. Grams,
I. Kullman, M. Bender and S. Goriely

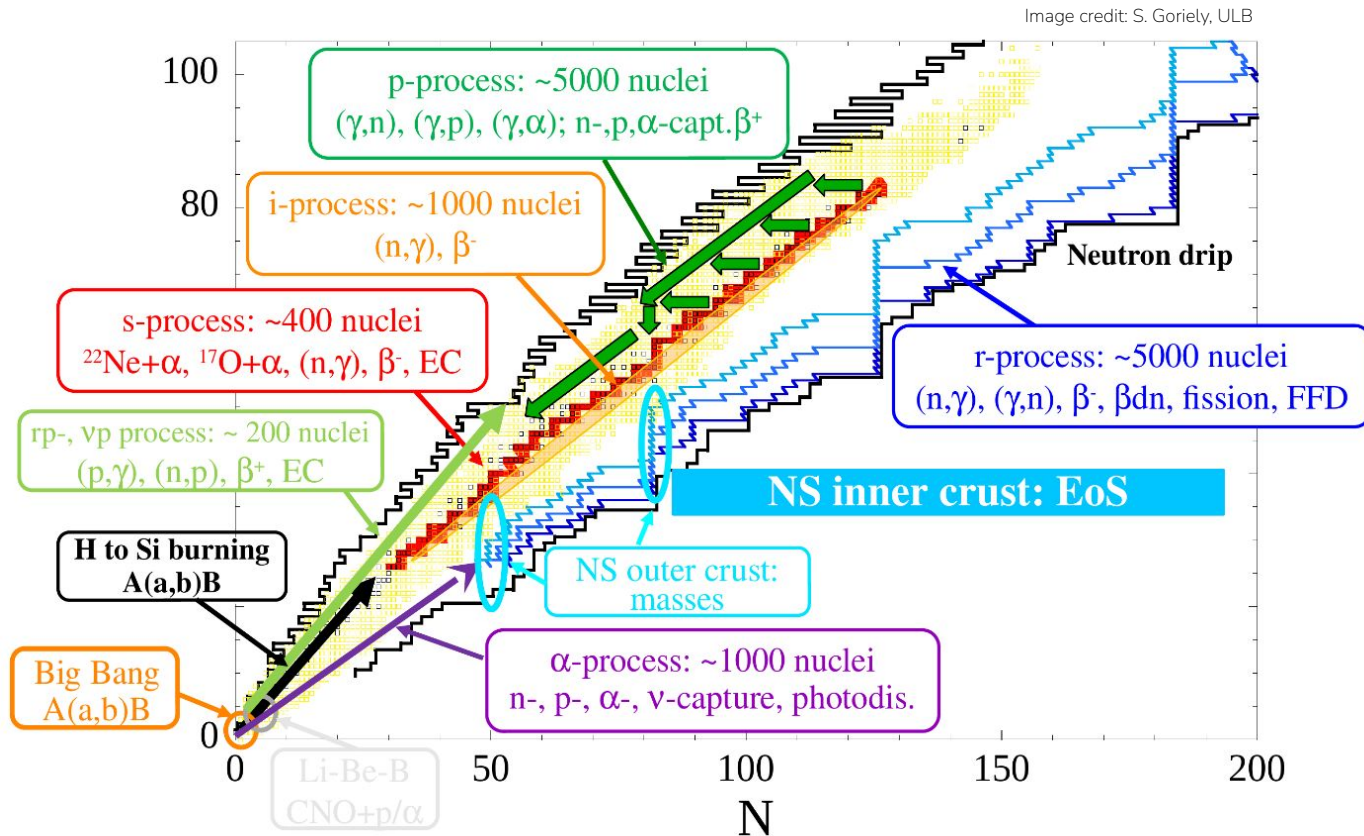
September 2022



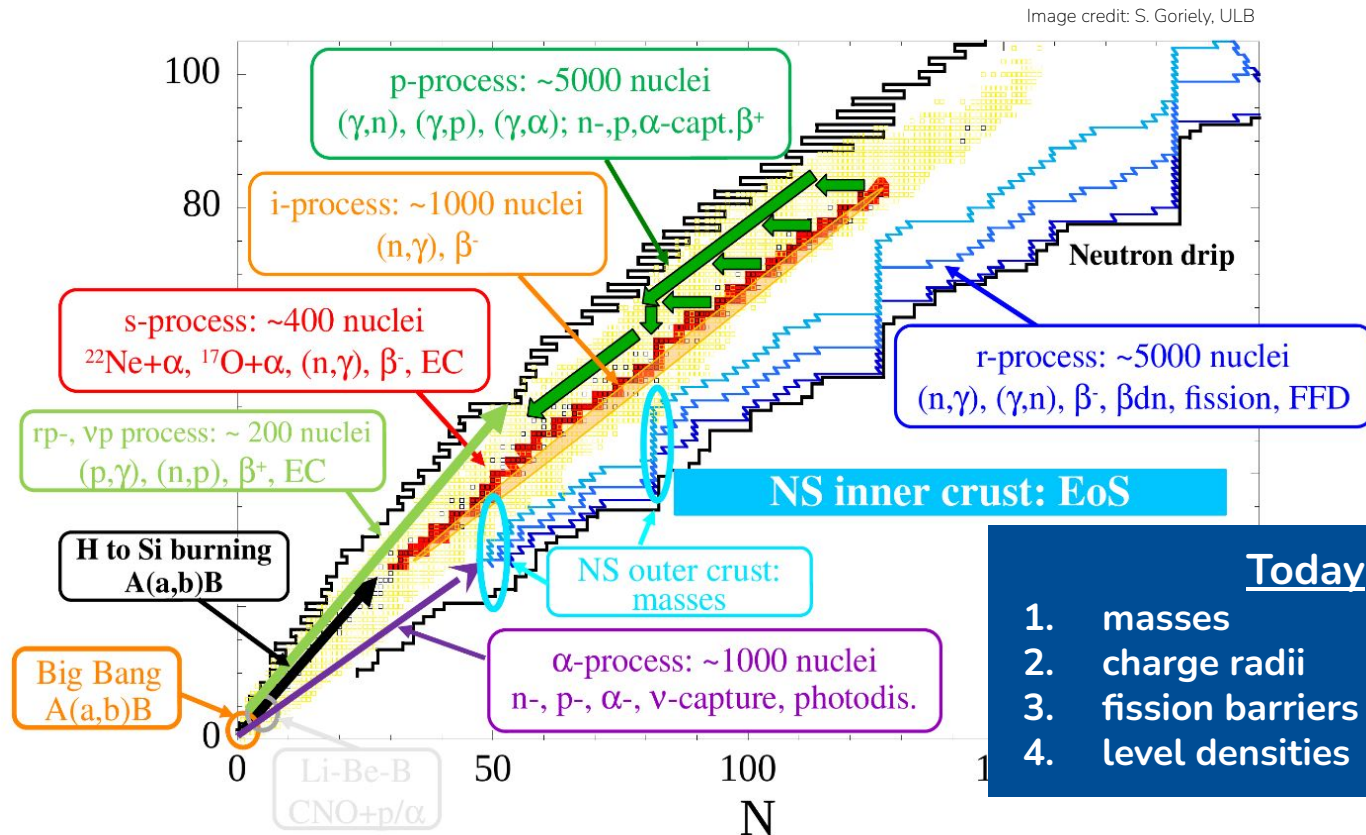
wryssens.com

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The challenge for nuclear theory



The challenge for nuclear theory



$$E \sim \int d^3r \left[C^\rho \rho(\mathbf{r})\rho(\mathbf{r}) + C^\tau \tau(\mathbf{r})\rho(\mathbf{r}) + \dots \right]$$



$$E \sim \int d^3r \left[C^\rho \boxed{\rho(\mathbf{r})\rho(\mathbf{r})} + C^\tau \boxed{\tau(\mathbf{r})\rho(\mathbf{r})} + \dots \right]$$

Local densities of a **HFB** state

Skyrme **E**nergy **D**ensity **F**unctionals (**EDFs**)

7

Coupling constants
(~ **25** parameters)

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Local densities of a **HFB** state

The diagram illustrates the Skyrme Energy Density Functional (EDF) equation. The equation is $E \sim \int d^3r [C^\rho \rho(\mathbf{r})\rho(\mathbf{r}) + C^\tau \tau(\mathbf{r})\rho(\mathbf{r}) + \dots]$. A blue line connects the coupling constants C^ρ and C^τ to the text 'Coupling constants (~ 25 parameters)'. A red line connects the density terms $\rho(\mathbf{r})\rho(\mathbf{r})$ and $\tau(\mathbf{r})\rho(\mathbf{r})$ to the text 'Local densities of a HFB state'.

Skyrme **E**nergy **D**ensity **F**unctionals (**EDFs**)

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Local densities of a **HFB** state

The diagram shows the Skyrme Energy Density Functional (EDF) equation. The energy E is represented by a magenta box. The integrand consists of two terms: $C^\rho \rho(\mathbf{r})\rho(\mathbf{r})$ and $C^\tau \tau(\mathbf{r})\rho(\mathbf{r})$. The coupling constants C^ρ and C^τ are enclosed in blue boxes, and the density terms $\rho(\mathbf{r})\rho(\mathbf{r})$ and $\tau(\mathbf{r})\rho(\mathbf{r})$ are enclosed in red boxes. A blue line connects the text 'Coupling constants (~ 25 parameters)' to the blue boxes. A red line connects the text 'Local densities of a HFB state' to the red boxes. The equation is preceded by a magenta line connecting to the text 'Energy'.

Skyrme **E**nergy **D**ensity **F**unctionals (**EDFs**)

Energy

Coupling constants
(~ **25** parameters)

Fitted to:

- **2457** masses
- **884** charge radii
- Infinite matter properties

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Local densities of a **HFB** state

Strong points

- starts from **effective interaction**
- **wave-function** with individual nucleons
- many **observables** accessible
- Feasible for **1000s** of nuclei

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Strong points

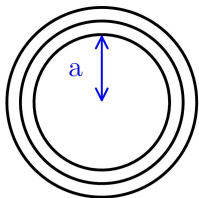
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Where does the physics enter?

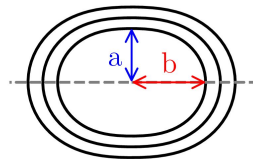
- Complexity of the **wave-function**
- Selection (and weighting) of **fitting data**
- **Functional form** of the EDF

Brussels models in 1-2-3 dimensions!

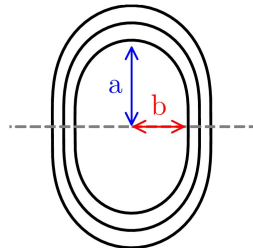
Spherical



Prolate



Oblate

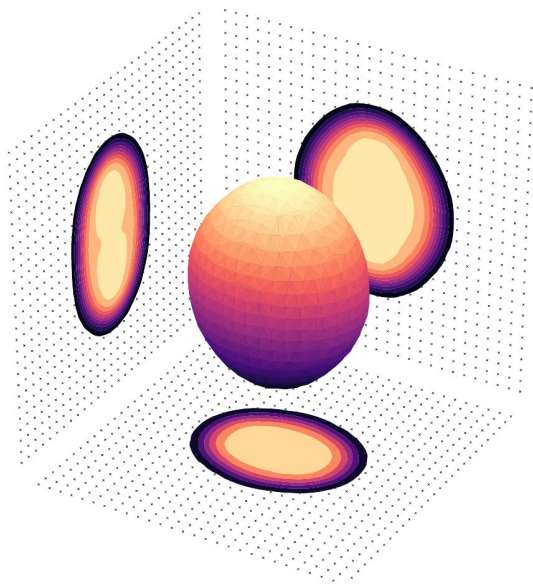


One DOF: β_{20}

Before: BSk1-32

- Axially symmetric
- Time-reversal invariant;
equal filling approximation
- Cheap to simulate!

Brussels models in 1-2-3 dimensions!



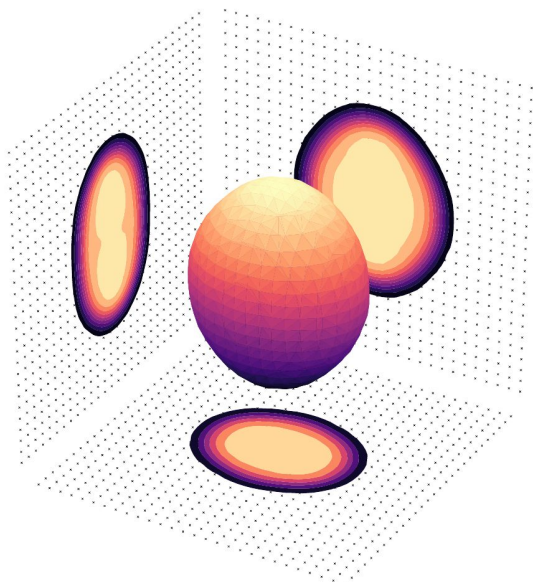
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Today: BSkG1,2

- 3D simulations
- no more equal filling

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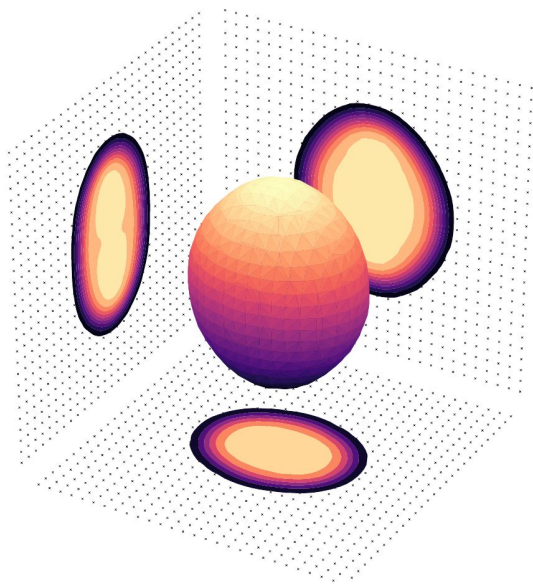
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W. R. PhD Thesis, ULB (2016).
G. Scamps et al., EPJA 57, 333 (2021).

- **3D** simulations
- **no more equal filling**
- MOCCa: r-space solver

Brussels models in 1-2-3 dimensions!

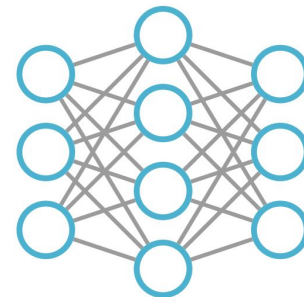


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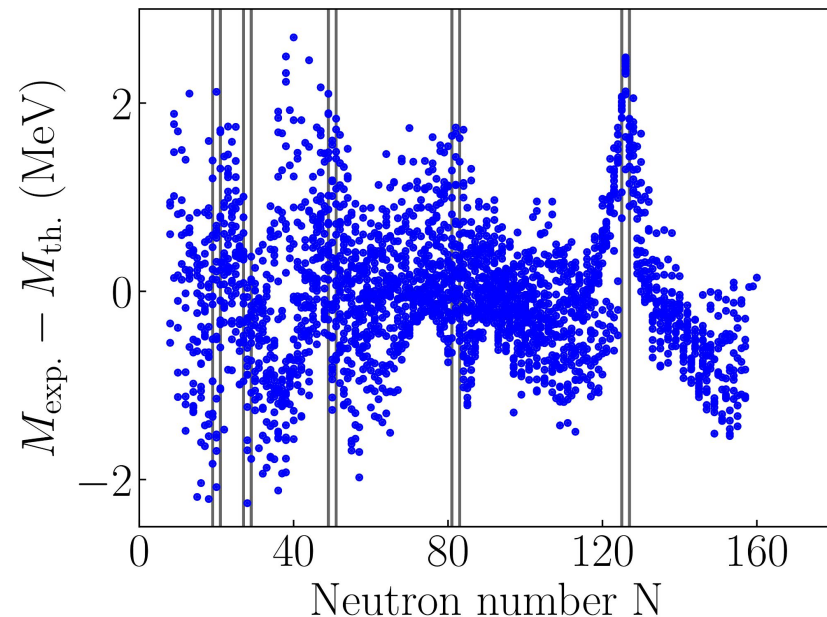
- **3D** simulations
- **no more equal filling**
- MOCCa: r-space solver
- **machine learning** to accelerate fit

BSkG1 & 2: the raw numbers

BSkG1:

- triaxial deformation

σ (MeV)	BSkG1	BSkG2	BSk14	FRLDM
2457 masses	0.741		0.729	0.808
884 charge radii (fm)	0.027		0.039	-
45 primary barriers				
45 secondary barriers				
28 Isomer energies				



BSkG2: W. Ryssens et al., [arXiv:2208.06455 \[nucl-th\]](https://arxiv.org/abs/2208.06455)

BSkG1: G. Scamps et al., EPJA 57, 333 (2021).

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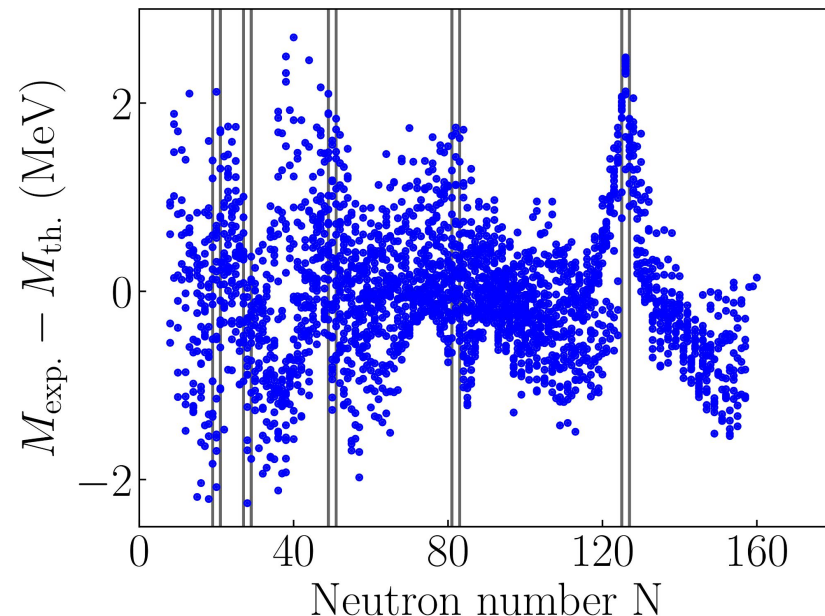
BSkG1:

- triaxial deformation

BSkG2:

- triaxial deformation
- time-reversal breaking
- Reference fission barriers put into the fit

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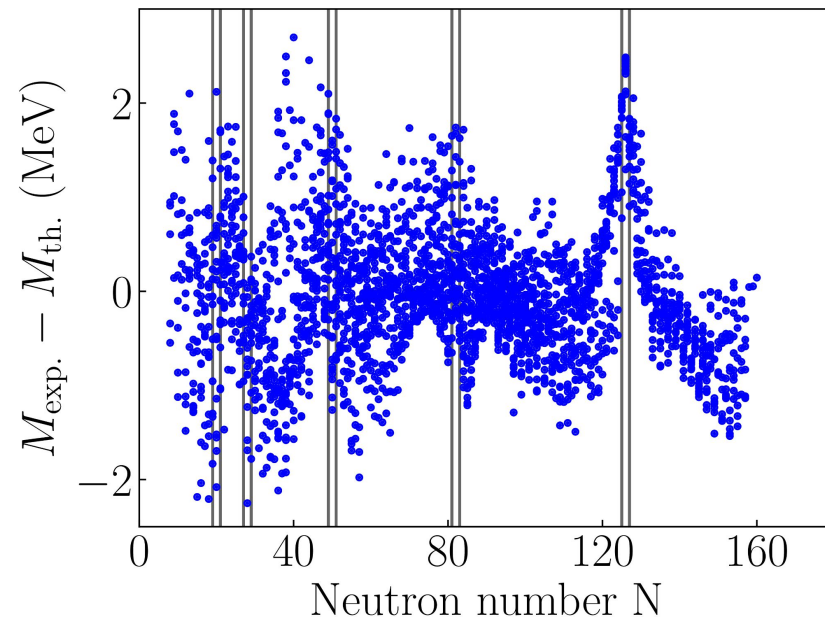
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884 charge radii (fm)	0.027	0.027	0.039	-
45 primary barriers	0.87	0.44	0.61	0.79
45 secondary barriers	0.86	0.47	0.70	1.35
28 Isomer energies	0.45	0.49	0.93	1.04



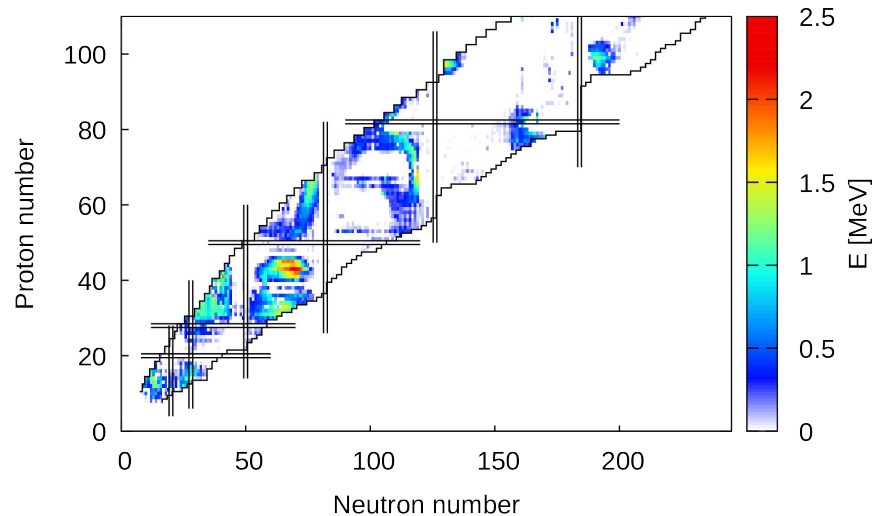
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What is the effect on **masses**?

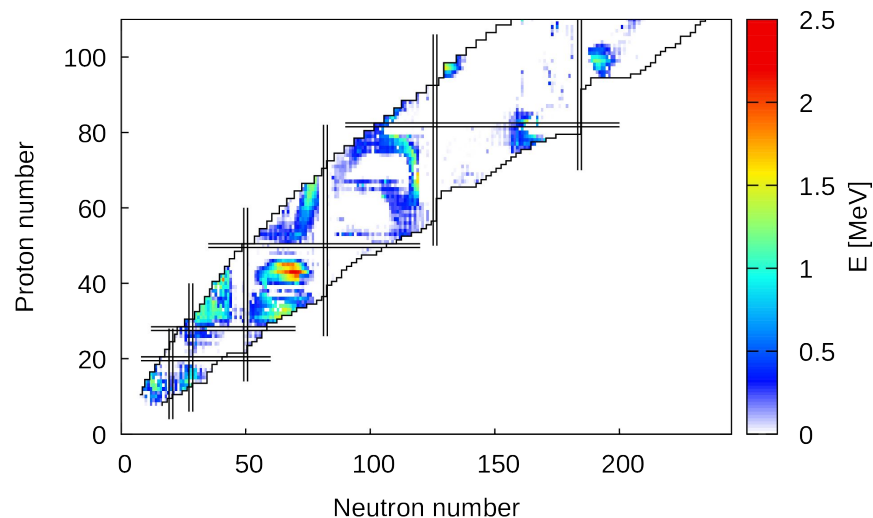


Triaxial deformation

G. Scamps et al., EPJA 57, 333 (2021).

- most nuclei remain axial
- several regions of triaxiality
- large effects from 0.5-2 MeV.

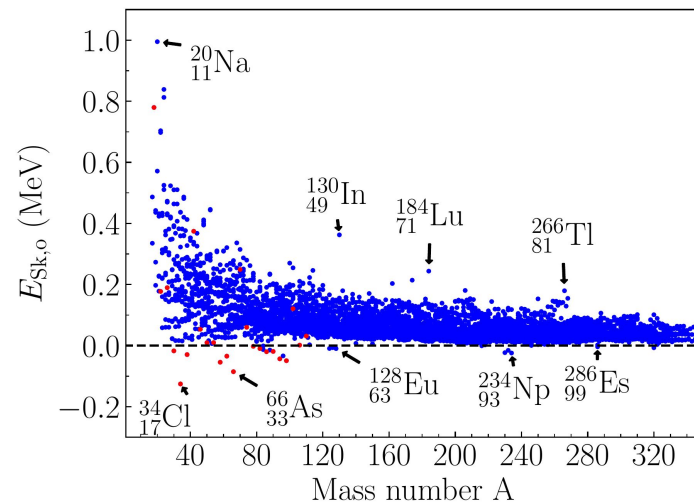
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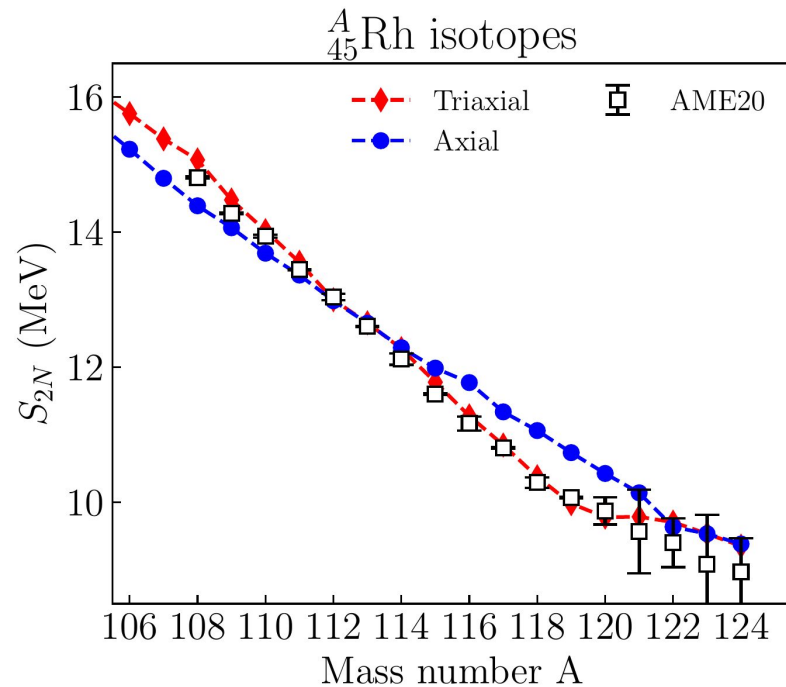


Time-reversal breaking

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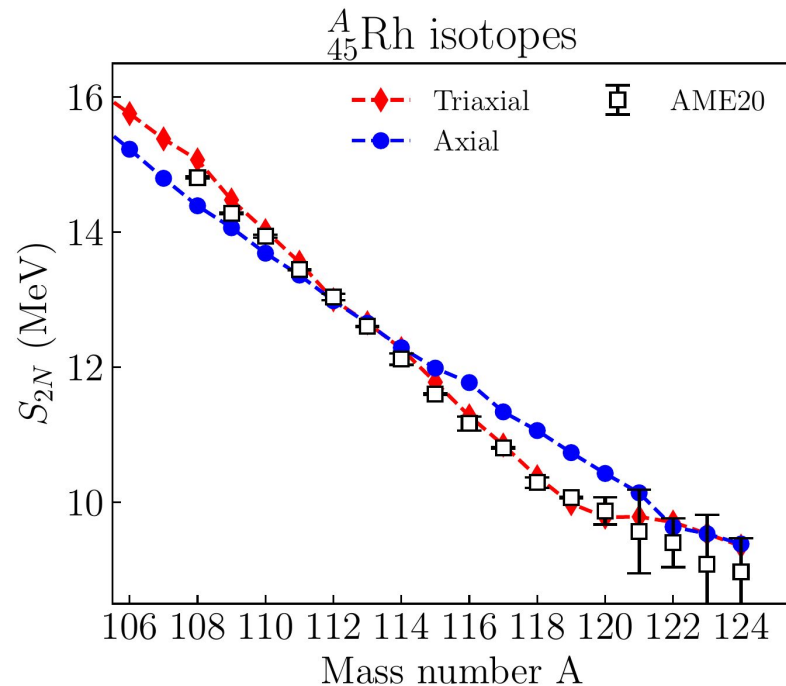
- included during fit, **no** simplifications
- reduces binding energies
- overall small effect

Detailed studies possible: **Z** $\approx$ 45.

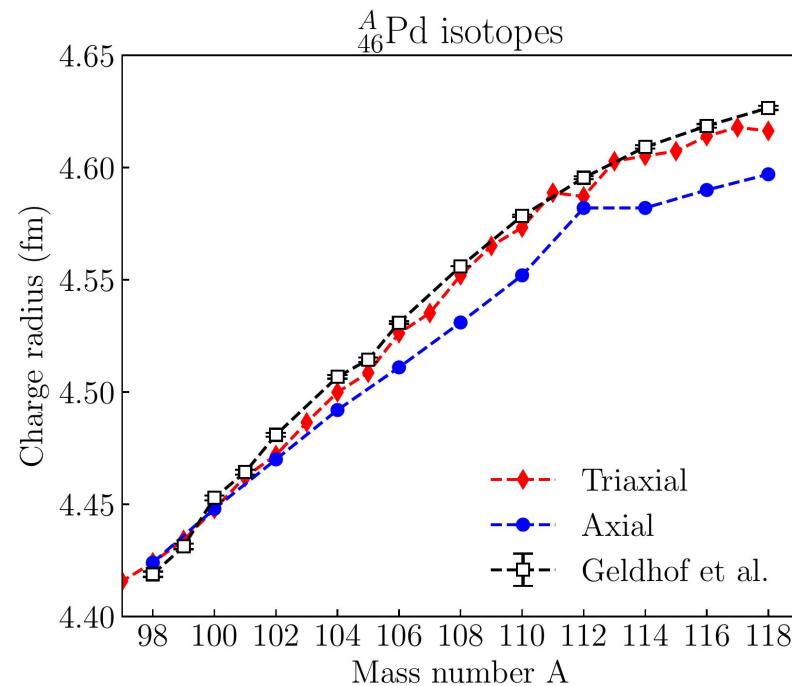


New data from JYFLTRAP: M. Hukkanen et al., in preparation.

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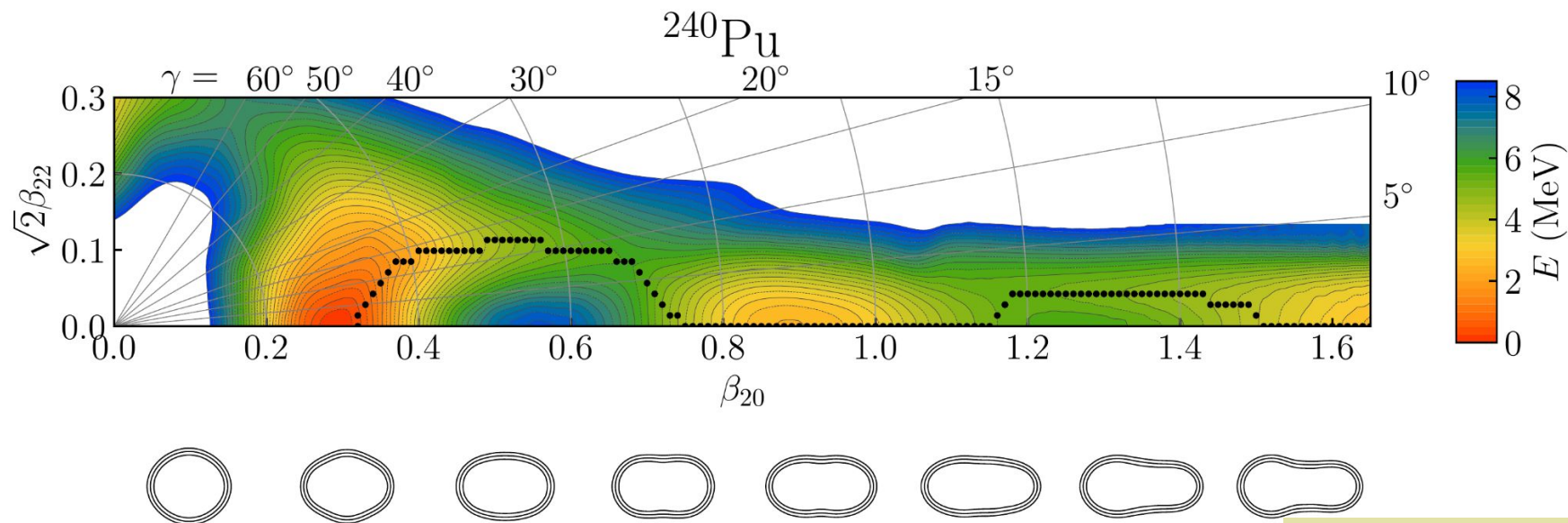


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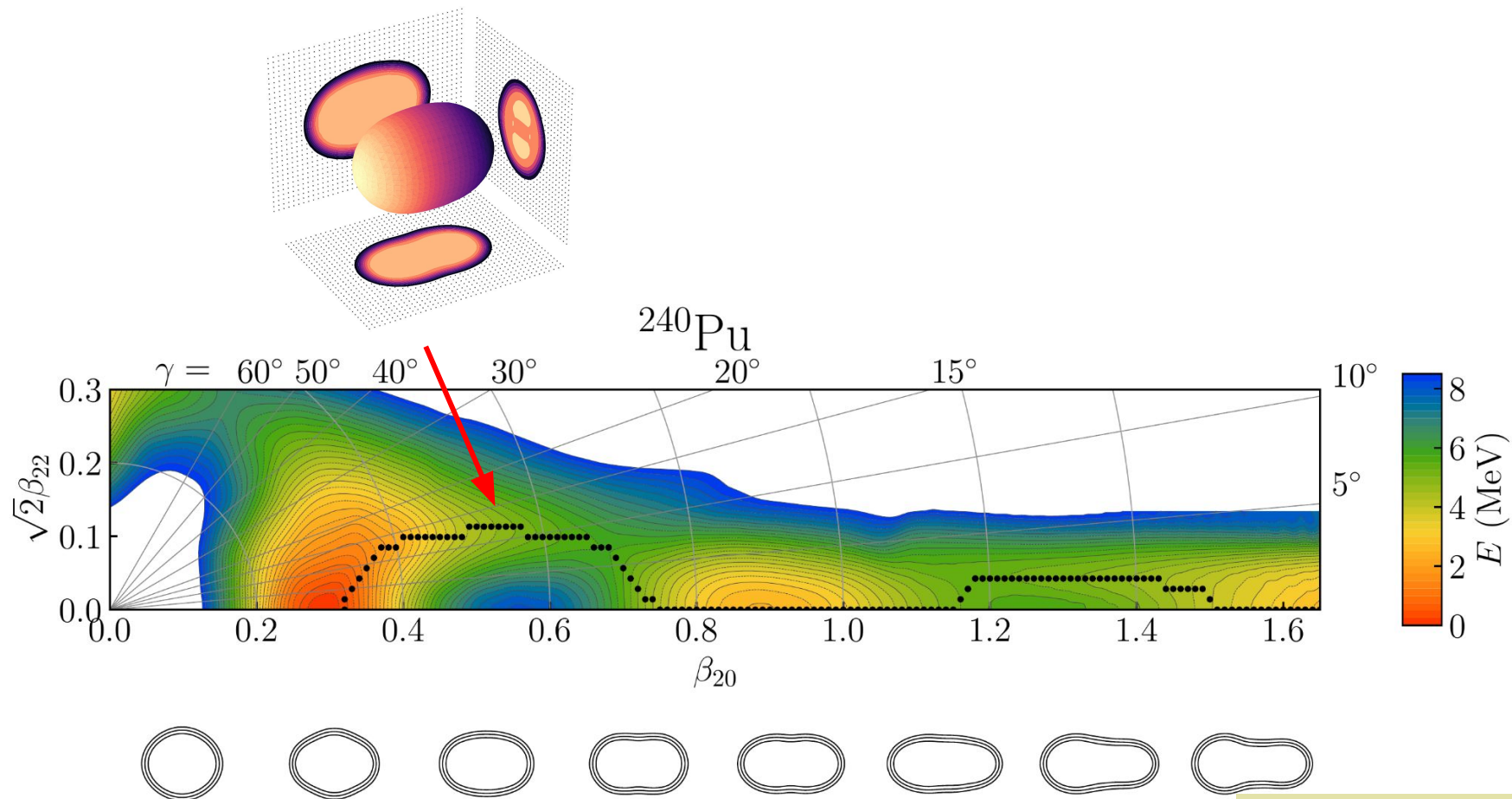


Data from S. Geldhof et al., PRL 128, 152501 (2022).

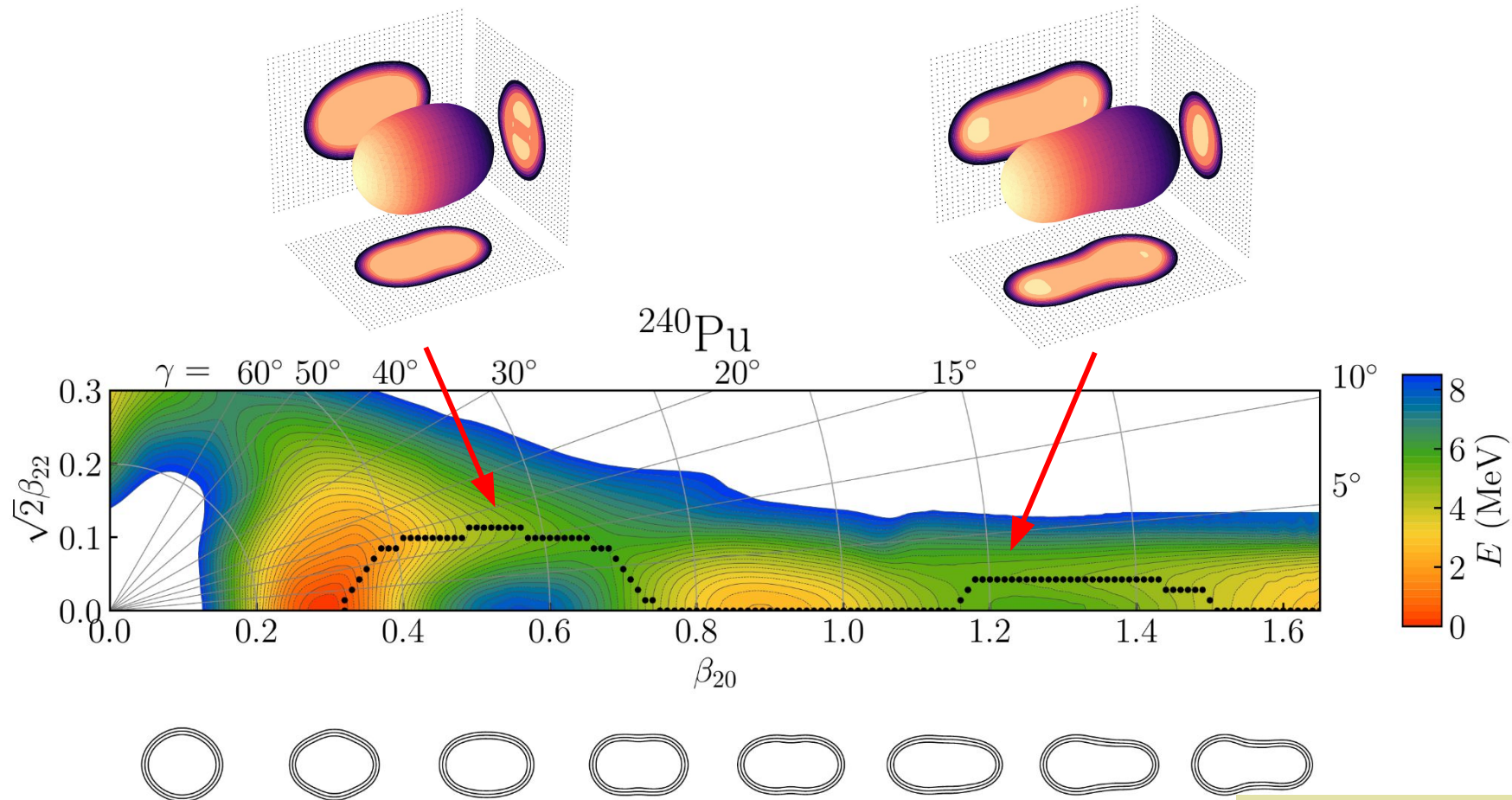
What is the effect on **fission barriers**?



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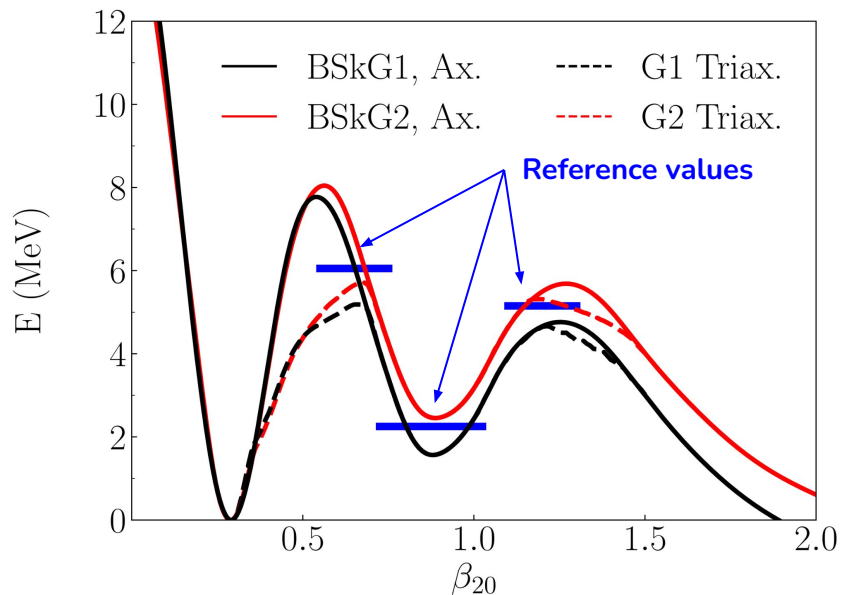


What is the effect on **fission barriers**?



Triaxiality has a **large** effect on barriers

Reference values from R. Capote et al., Nuclear Data Sheets **110**, 3107 (2009).

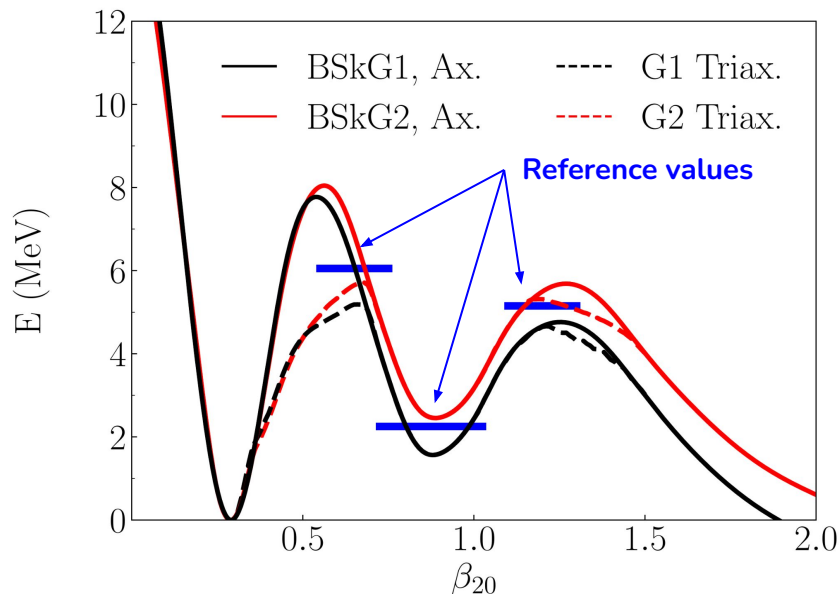


Triaxial deformation for ^{240}Pu

- Large effect on inner barrier
- No effect on isomers
- Modest effect on outer barrier

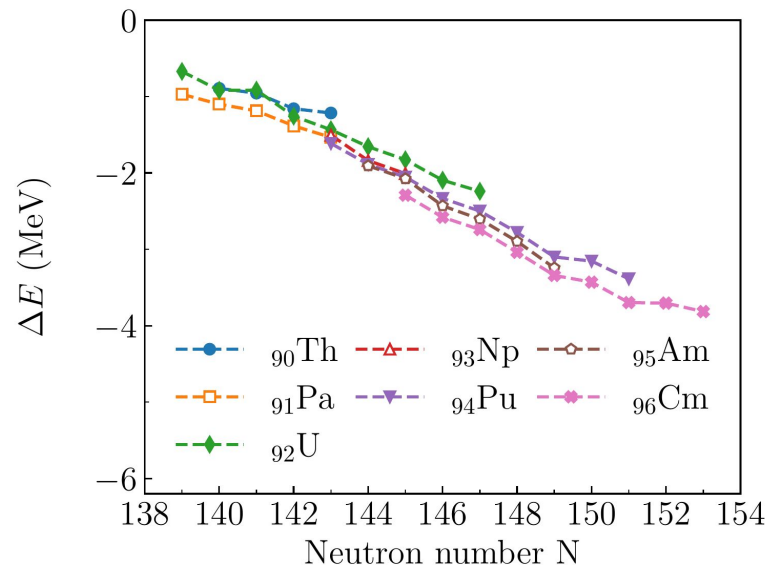
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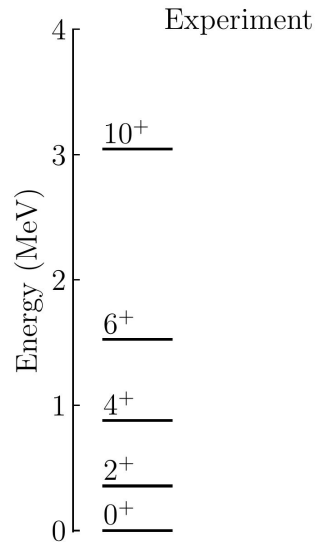
Triaxial deformation for actinides

- Larger effects with growing N
- reminder: $\sigma(\text{fission}) < 0.5 \text{ MeV}$
- what other regions does it affect?

What is the effect on **nuclear level densities**?



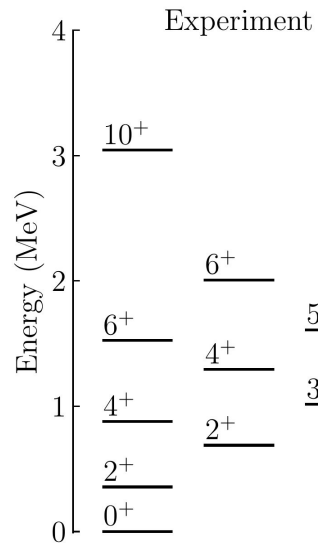
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Broken symmetries impact NLDs

- **axial** rotors give rise to **sparse** spectra

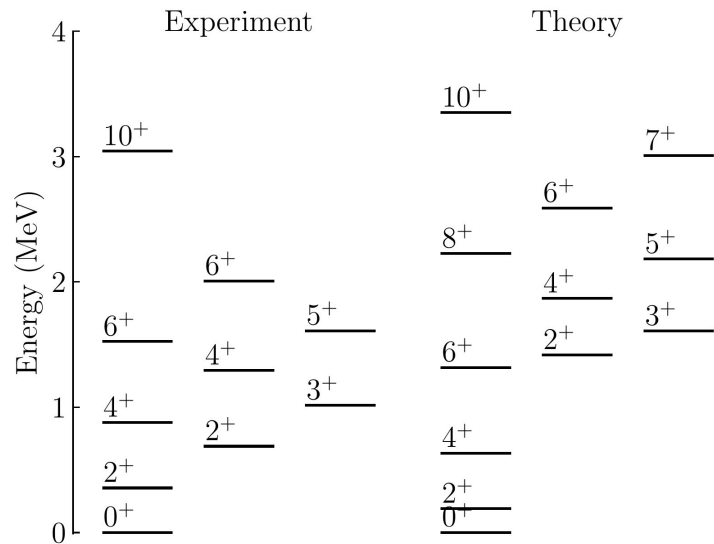
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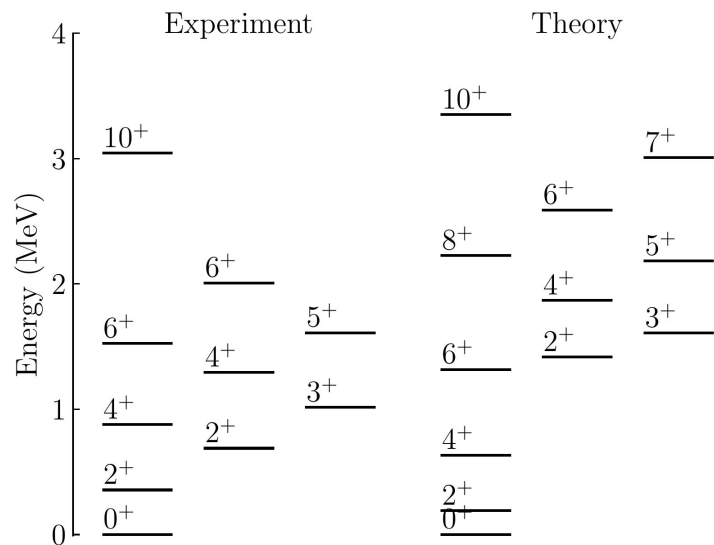
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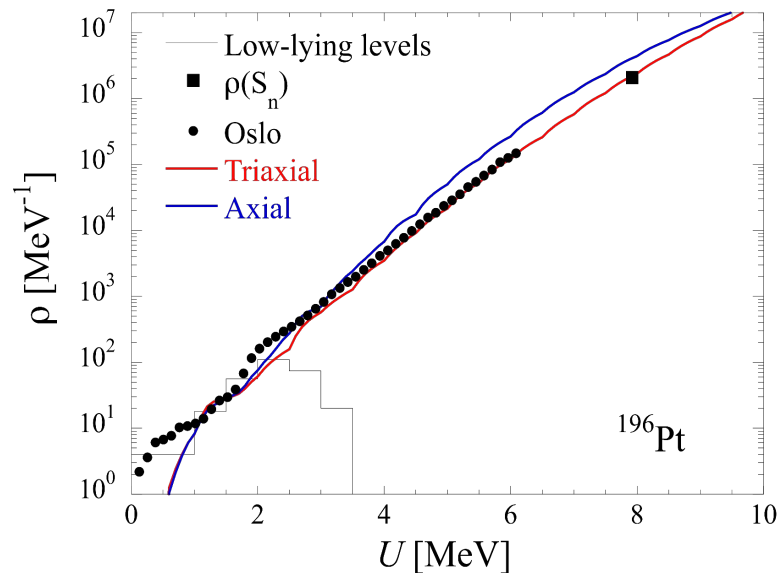
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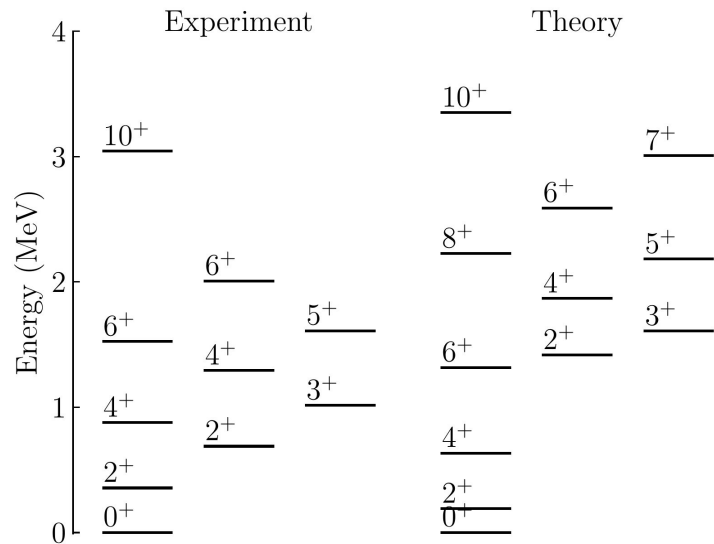


Level densities with BSkG2

F. Giacoppo et al., PRC 90, 054330 (2014).

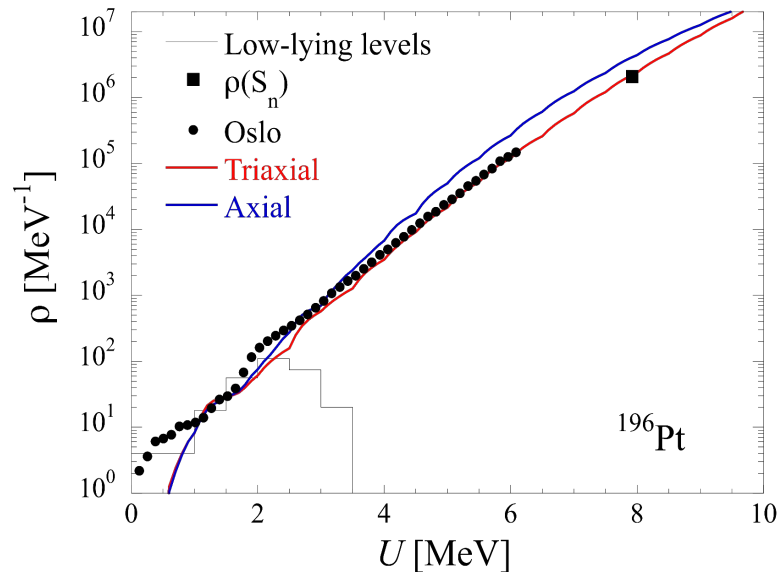
- not always higher level density
- but a **different energy dependence!**

What is the effect on **nuclear level densities**?



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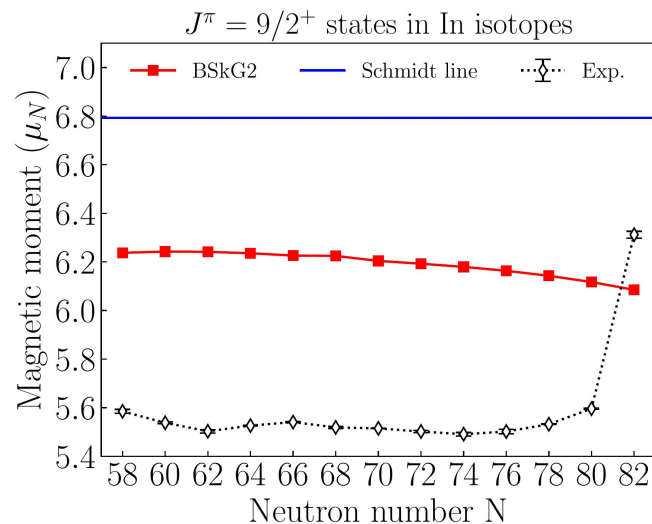
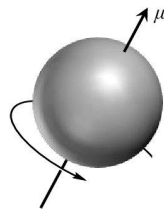


Level densities with BSkG2

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- not always higher level density
- but a **different energy dependence!**
- systematic calculations underway

The future: **new observables**

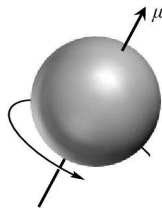
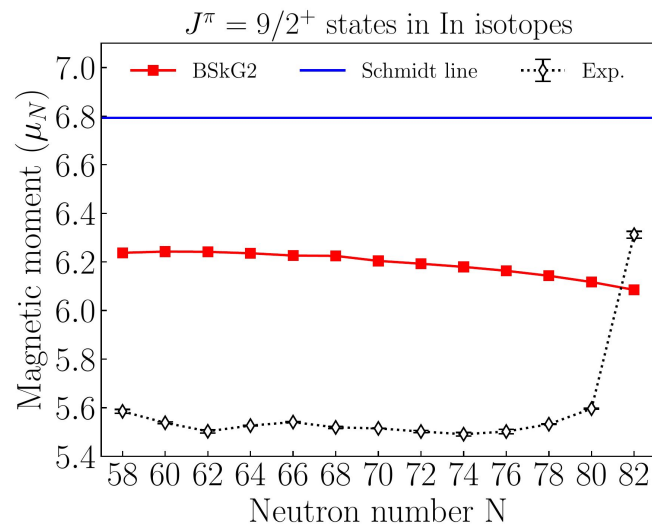


A. R. Vernon et al., Nature 607, 260 (2022),
J. Eberz et al., NPA 464, 9 (1987).

Magnetic moments

- time-reversal breaking needed!
- BSkG2 produces part of polarisation

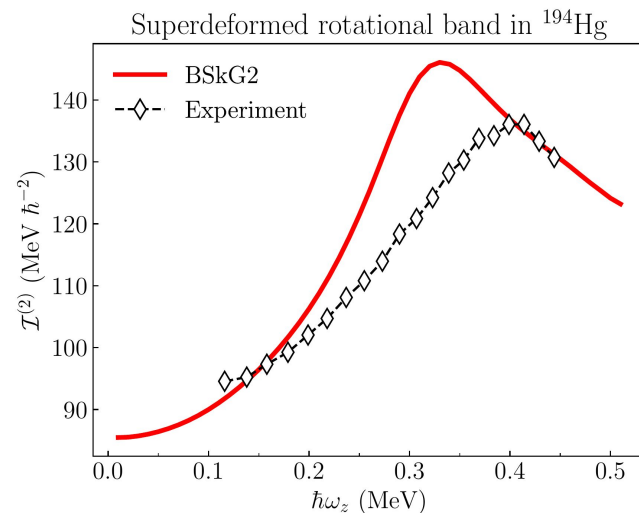
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Magnetic moments

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Rotational bands

B. Singh et al., Nuclear Data Sheets **97**, 241 (2002).

- time-reversal breaking needed!
- BSkG2 reproduces MOI decently well

The future: **neutron stars**

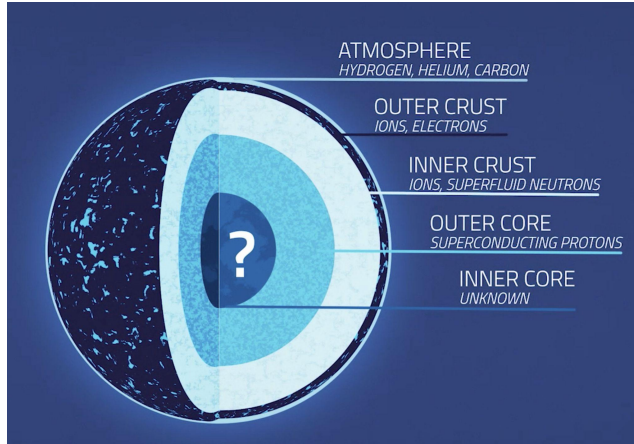


Image credit: NASA conceptual image lab

The future: **neutron stars**

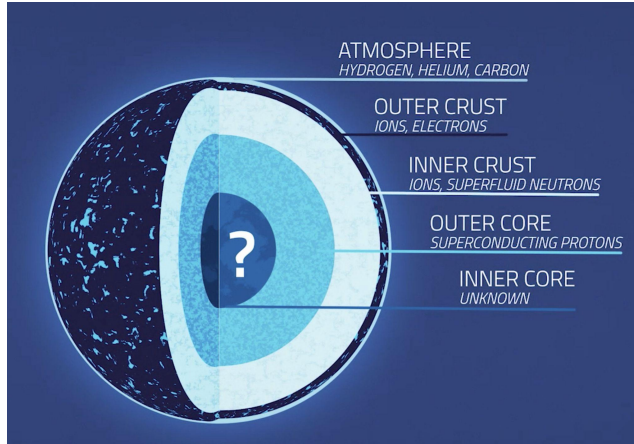
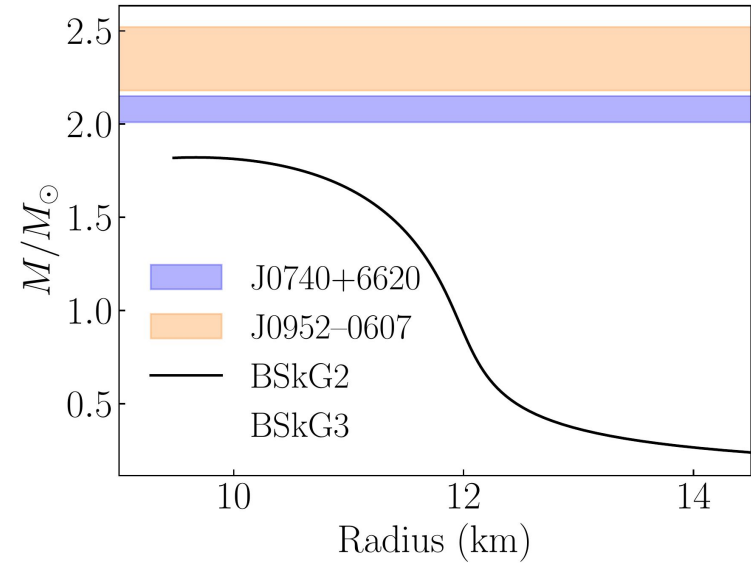


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E. Fonseca *et al.*, *ApJL* **915**, L12 (2021) ; R. W. Roman *et al.*, *ApJL* **934**, L17 (2022).

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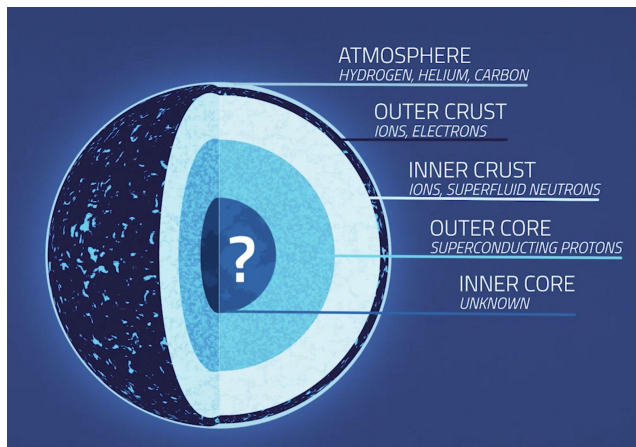
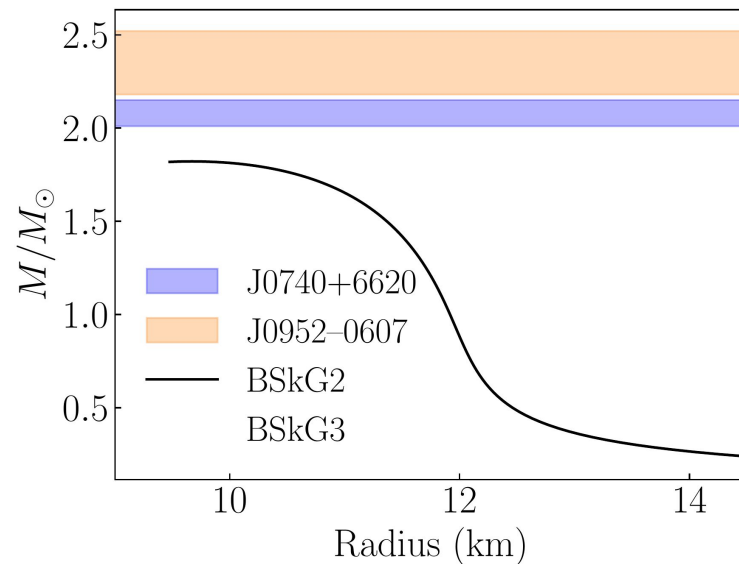


Image credit: NASA conceptual image lab



Extending the functional form

- microscopically motivated pairing
- extra density dependencies in the EDF

E. Fonseca *et al.*, *ApJL* **915**, L12 (2021) ; R. W. Roman *et al.*, *ApJL* **934**, L17 (2022).

The future: **neutron stars** and **BSkG3**

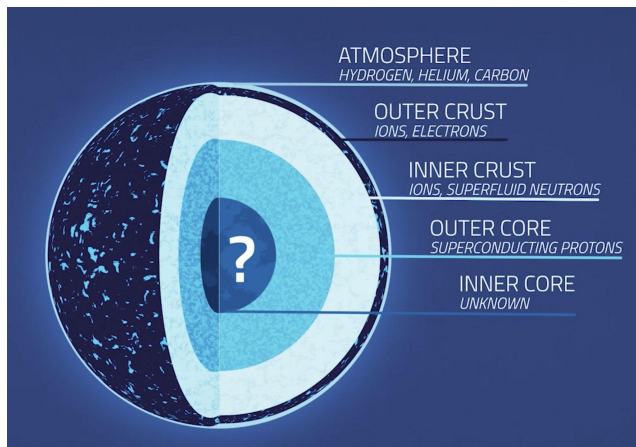
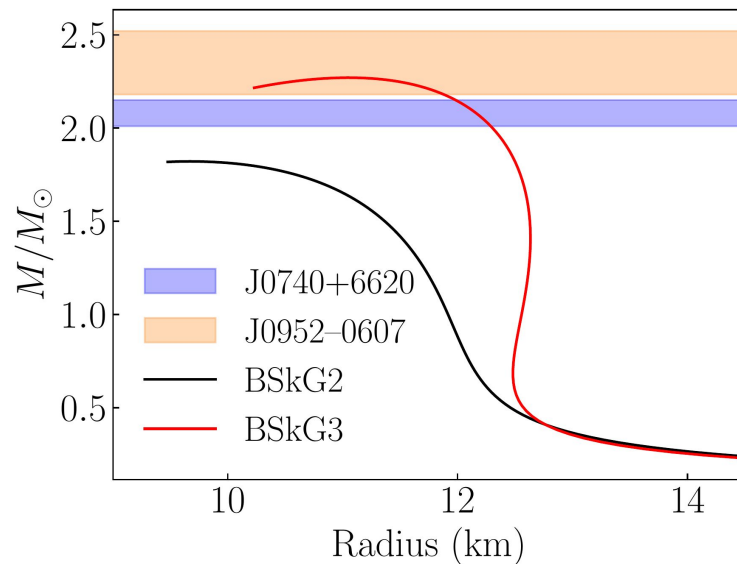


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Extending the functional form

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BSkG3

E. Fonseca *et al.*, ApJL **915**, L12 (2021) ; R. W. Roman *et al.*, ApJL **934**, L17 (2022).

- as good for finite nuclei as BSkG2...
- ...but able to support massive pulsars
- See our poster by G. Grams *et al.*!

Conclusion

We provide **data for applications** that

- depends on a **limited number of parameters**
- incorporates the **maximum** of nuclear structure **physics**
- with a **focus on absolute masses...**
- but not at the cost of **other observables** (radii, barriers, ...)

BSkG1: G. Scamps et al., EPJA 57, 333 (2021).
BSkG2: W. Ryssens et al., arXiv:2208.06455 [nucl-th]

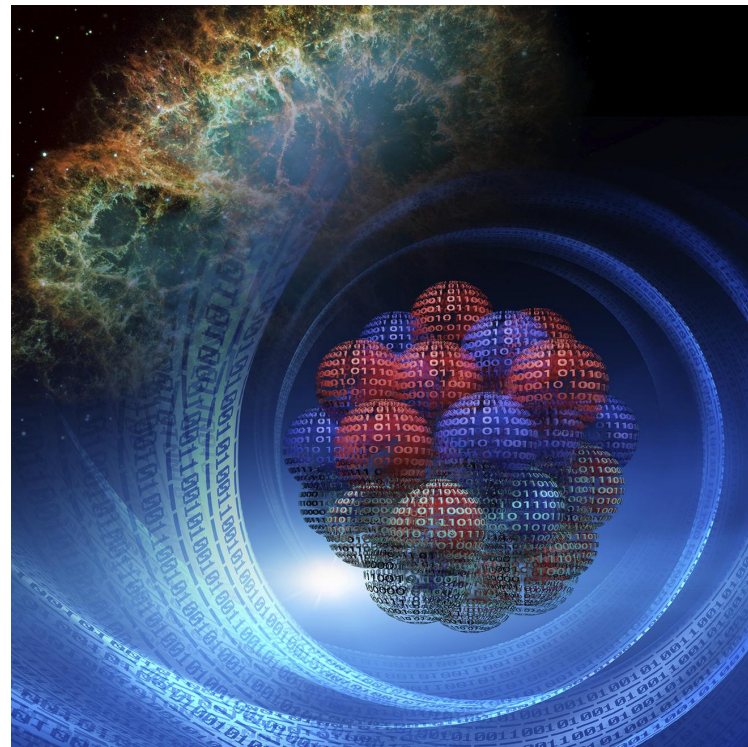


Image credit: Andy Sproles, ORNL

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The new **BSkG** models push further:

- **Triaxial deformation:** ground states, barriers, NLD's,...
- **Time-reversal breaking**

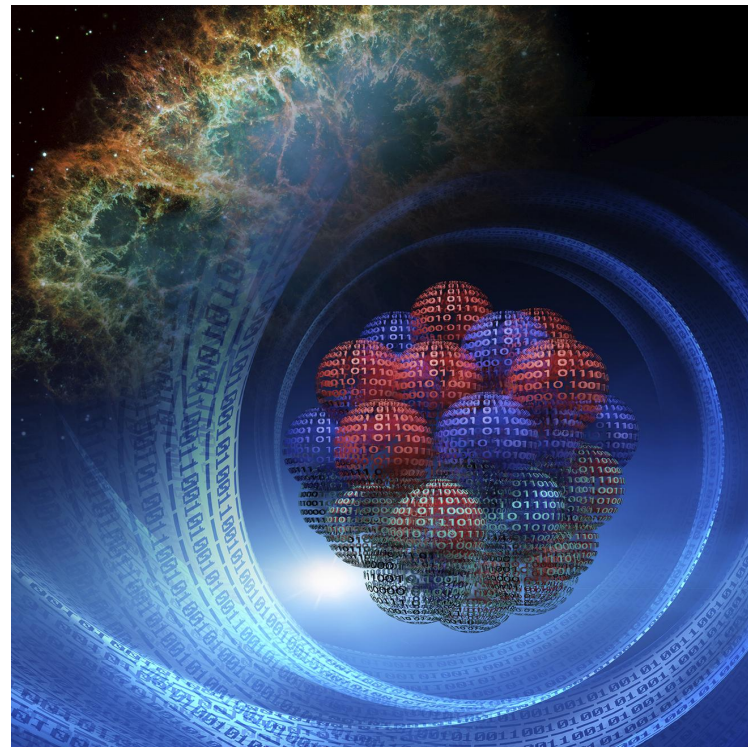


Image credit: Andy Sproles, ORNL

BSkG1: G. Scamps et al., EPJA 57, 333 (2021).

BSkG2: W. Ryssens et al., arXiv:2208.06455 [nucl-th]

Conclusion

We provide **data for applications** that

- depends on a **limited number of parameters**
- incorporates the **maximum** of nuclear structure **physics**
- with a **focus on absolute masses...**
- but not at the cost of **other observables** (radii, barriers, ...)

The new **BSkG** models push further:

- **Triaxial deformation:** ground states, barriers, NLD's,...
- **Time-reversal breaking**

BSkG2 describes **at the same time:**

- $\sigma(\text{masses}) < 0.8 \text{ MeV}$
- $\sigma(\text{fission}) < 0.5 \text{ MeV}$
- $\sigma(\text{radii}) < 0.03 \text{ fm}$

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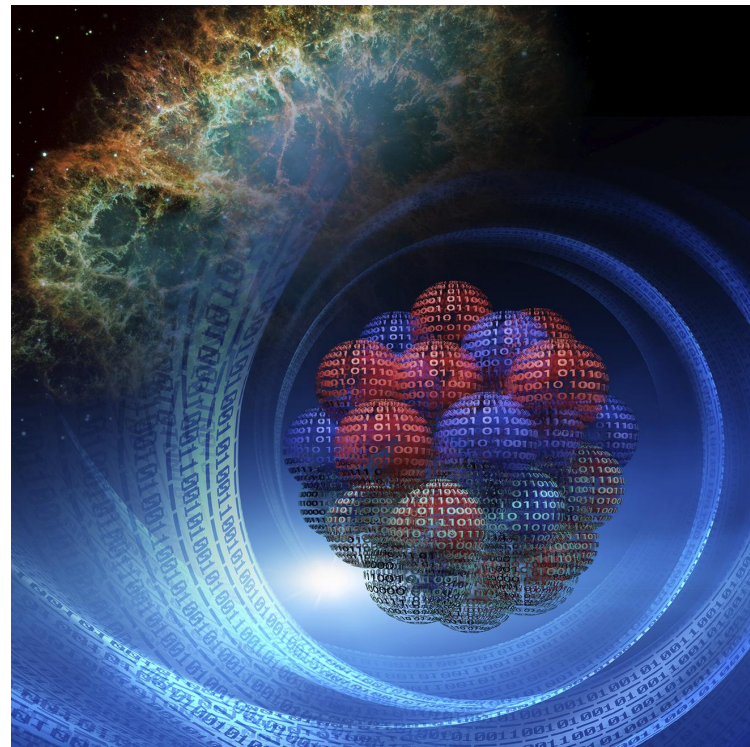


Image credit: Andy Sproles, ORNL

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Coming up:

- **Level density** tables for thousands of nuclei
- **Fission** tables for thousands of nuclei
- **BSkG3:** improved neutron matter EOS.

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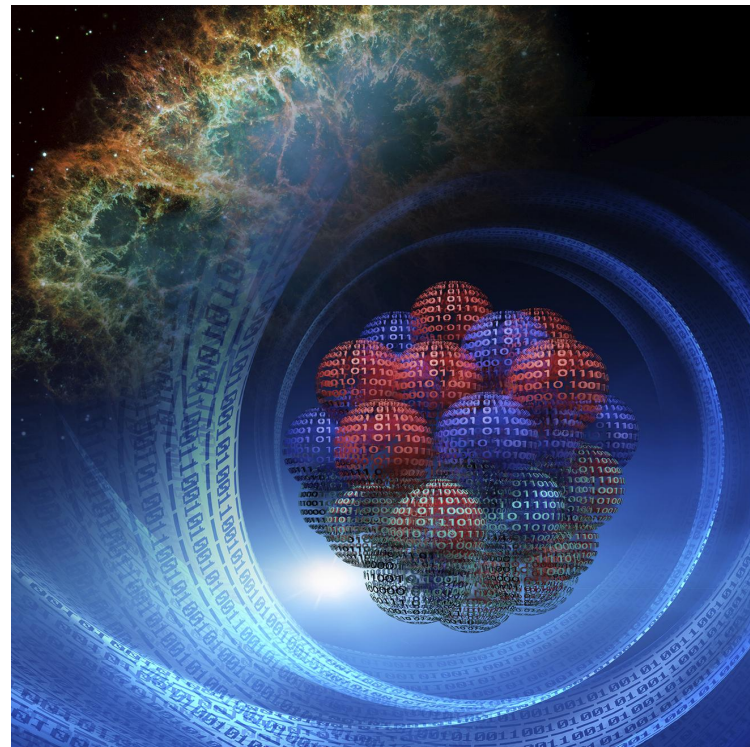


Image credit: Andy Sproles, ORNL

Thank you for...

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