

Symmetry breaking on the scale of the nuclear chart

W. Ryssens, G. Scamps, M. Bender, & S. Goriely

3rd of December 2021



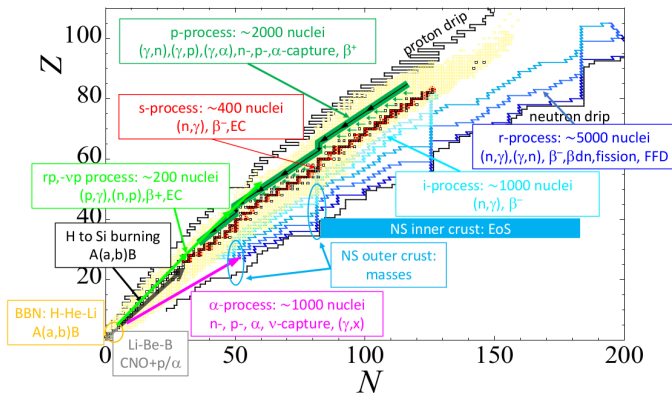
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Nuclear data for astrophysics



M. Arnould & S. Goriely, Prog. Part. Nuc. Phys. **112**, 103766 (2020).

Energy density functionals (EDFs)

$$E \sim \int d^3r \left[C^\rho \rho(\vec{r})\rho(\vec{r}) + C^\tau \rho(\vec{r}) \tau(\vec{r}) + \dots \right]$$

- Pheno. form and fit of C 's
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- Fitted on $\lesssim 10$ nuclei
- Typically ~ 15 parameters
- **Advanced** calculations

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We are bringing the "structure" to the "astro" scale!

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Also Gogny (**D1M**, 2009) and relativistic EDFs (**DD-MEB1/2**, 2016)!

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	Old		New	
	BSk27	BSk32	BSkG1	BSkG2
Parameters	24	33	22	25
$\sigma(M)$ [MeV]	0.512	0.529		
$\sigma(S_n)$ [MeV]	0.425	0.489		
$\sigma(Q_\beta)$ [MeV]	0.516	0.601		
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Skyrme form	Standard	Extended		
Micr. Pairing	No	Yes		
EOS n-matter	Soft	Stiff		
Deformation	Axial	Axial		
Representation	Oscillator	Oscillator		
	2D	2D		
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Status	Published	Published		

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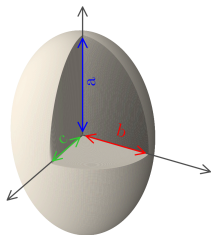
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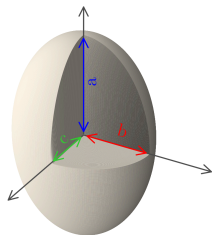
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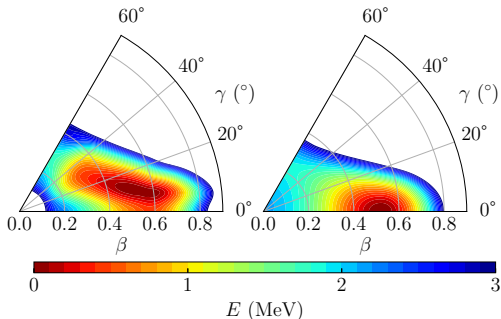
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BSkG1 vs. BSkG2

$$\text{BSkG2} = \text{BSkG1} + \text{time-odd terms} + \text{vibrational correction}$$

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- Many different approaches exist
- “Simplest” possible form
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- Blocking calculations with polarisation!

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$$E_{\text{vib}} = -\textcolor{red}{d} \sum_{\mu=x,y,z} B_\mu e^{-l(B_\mu - \textcolor{red}{B}_0)^2} \frac{\langle \hat{J}_\mu^2 \rangle}{2\mathcal{I}_\mu},$$

- Three additional parameters
- Refit on 12 even-even RIPL-3 fission barriers

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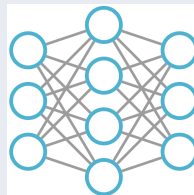
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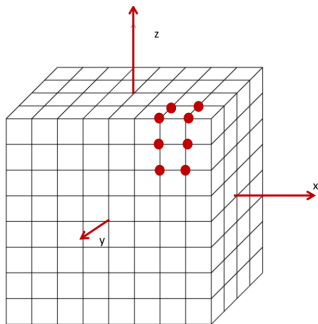
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Machine learning



- 100s of Neural Networks
- Keras implementation
- Learning parameter space

MOCCa: speed & accuracy



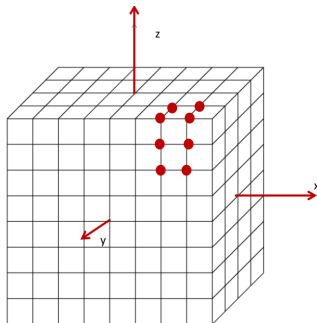
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- 10s of keV on energies
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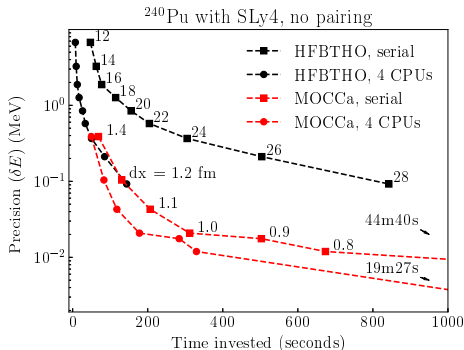
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Better algorithms for speed

- factor 10 over EV8
- **human-tweaking free**

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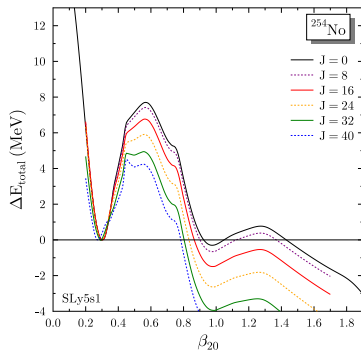
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Algorithms for constraints

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- Many **simultaneous** constraints
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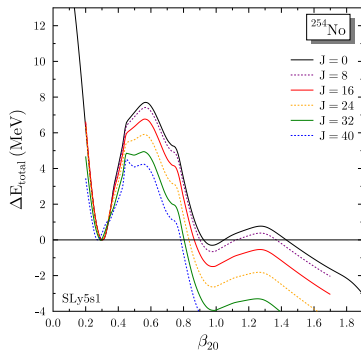


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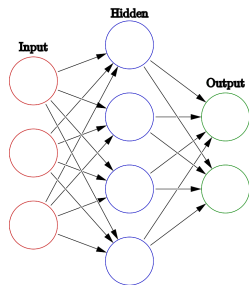
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Blocked HFB $|\Psi\rangle$ with gradient solver

- lowest E **guaranteed**
- 99% convergence rate
- **human-tweaking free**

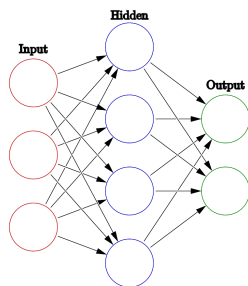
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Neural networks



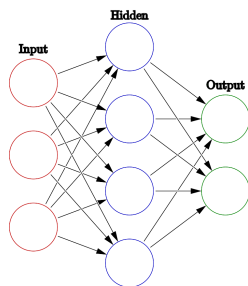
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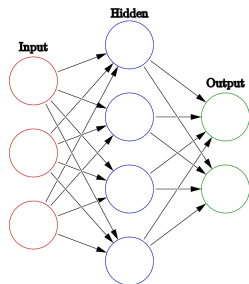
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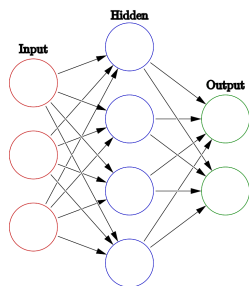
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AI in nuclear physics

R. Lasserri et al., PRL 124, 162502 (2020).
N. Schunck et al., EPJA 51, 169 (2015).
N. Schunck et al., J. Phys. G 47, 10 (2020).
Z. M. Niu and H.Z. Liang, PLB 778, 48 (2018).

L. Neufcourt et al., PRC 98, 034318 (2018).
E. Yüksel et al., IJMP E30, 2150017 (2021).
Z. Gao et al., arXiv 2105.02445 (2021).
... [many more]...

Replacing MOCCa by a neural network

Input

- N, Z
- model params

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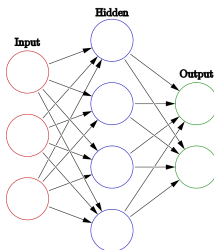
Output

- Energy
- Charge radius
- Deformation
- ...

Replacing MOCCa by a neural network

Input

- N, Z
- model params
- $A^{2/3}, (N - Z)^2/A, \dots$
- magic numbers



Output

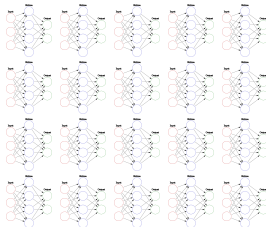
- Energy
- Charge radius
- Deformation
- ...

- 4 hidden layers
- 128/64/32/16 neurons
- **12289** parameters
- necessarily biased

Replacing MOCCa by 300 neural networks

Input

- N, Z
- model params
- $A^{2/3}, (N - Z)^2/A,$
...
- magic numbers

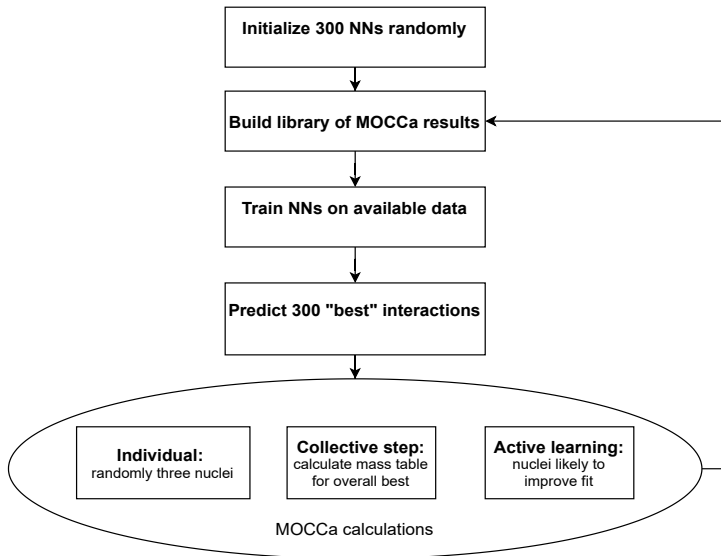


Output

- Energy
- Charge radius
- Deformation
- ...

- 4 hidden layers
- 128/64/32/16 neurons
- 12289 parameters
- Power in numbers

Fitting algorithm



Fitting algorithm

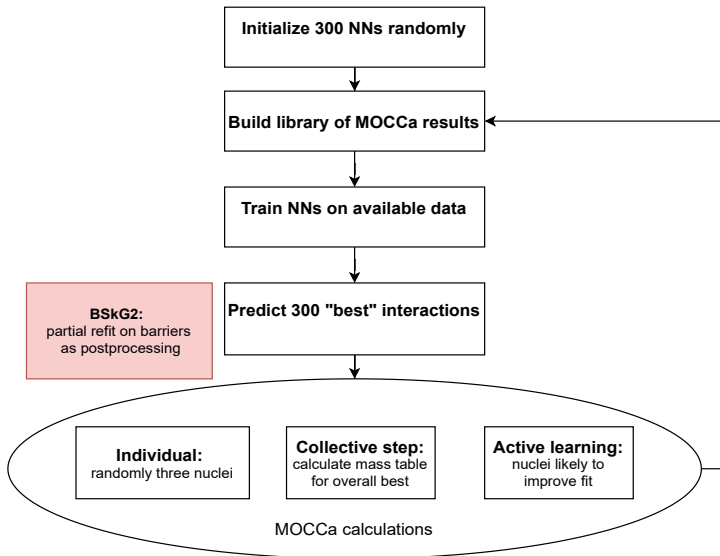


Table of contents

- 1 Introduction
- 2 The models: BSkG1/2
- 3 Tools
 - MOCCa
 - Machine learning
 - Fitting the BSkGX
- 4 Results
 - Nuclear masses and mass differences
 - Deformations
 - Fission barriers
 - Time-reversal breaking
- 5 Conclusions and outlook

All results shown were obtained with a **full** MOCCa calculation.

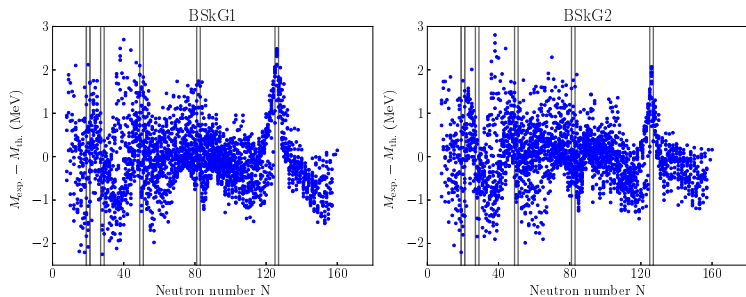
Clarification

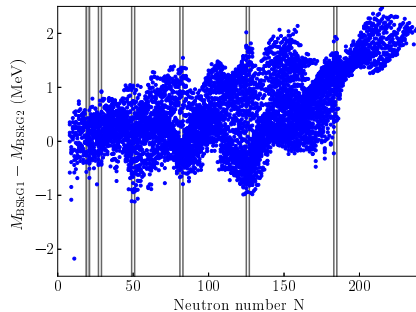
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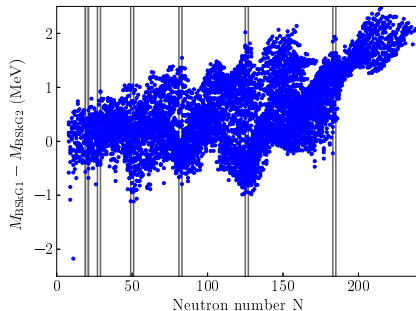
None of the results shown have been extra/interpolated with machine learning.

All results shown were obtained with a **full** MOCCa calculation.

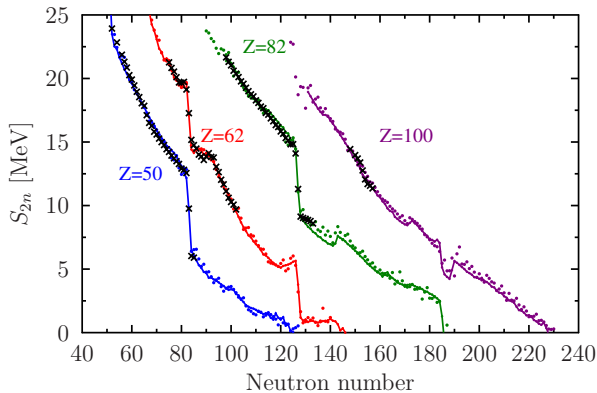
None of the results shown have been extra/interpolated with machine learning.
Machine learning is **only** used to perform the interaction fitting.



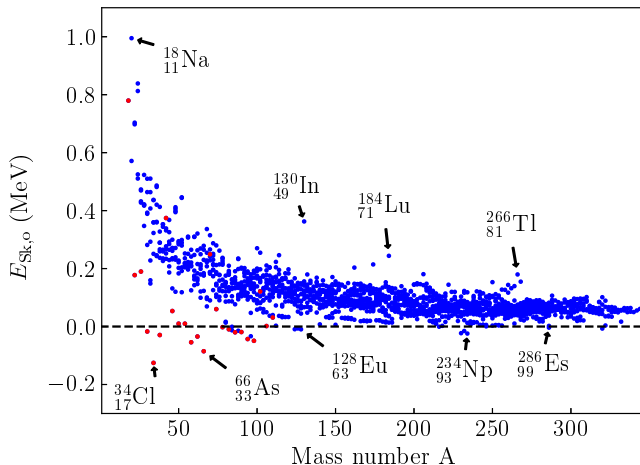




	Old		New	
	BSk27	BSk32	BSkG1	BSkG2
Parameters	24	33	22	25
$\sigma(M)$ [MeV]	0.512	0.529	0.741	0.678
$\sigma(S_n)$ [MeV]	0.425	0.489	0.466	0.500
$\sigma(Q_\beta)$ [MeV]	0.516	0.601	0.645	0.619
$\sigma(R_c)$ [fm]	0.028	0.027	0.027	0.027

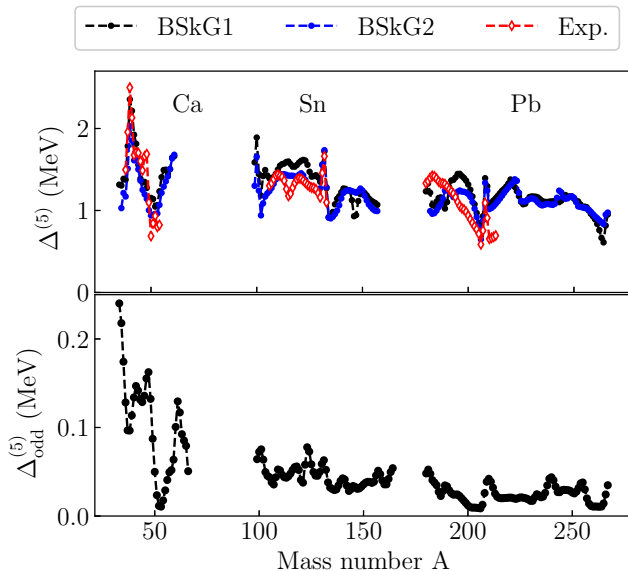


Time-odd energies

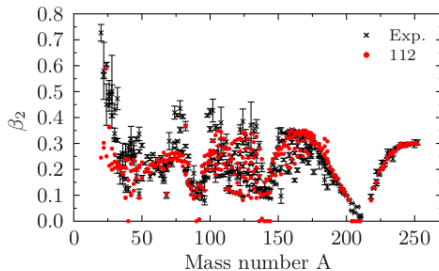
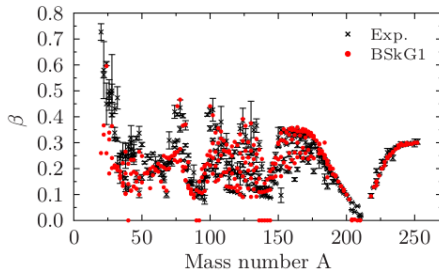


- Virtually all time-odd energies are positive ($\neq$ standard!)
- For $N = Z$ the effect can be negative
- Particularly large effects just outside shell closures

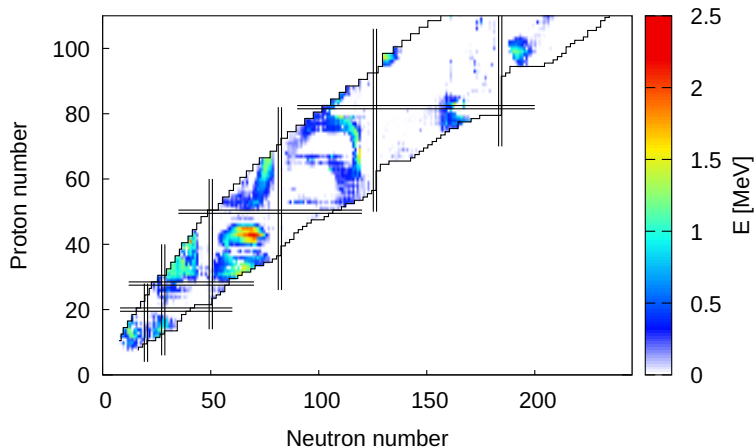
Mass differences



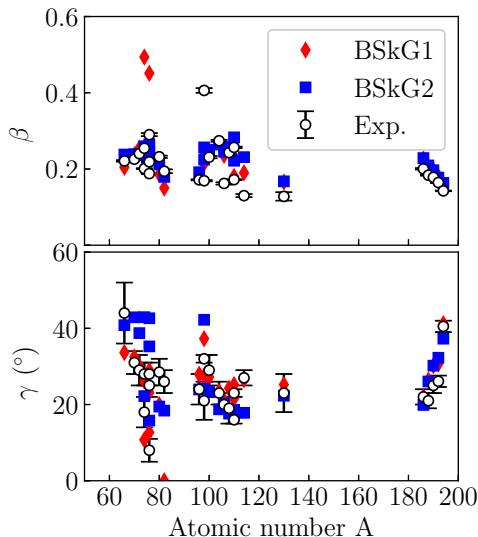
Deformations



Triaxiality: where?



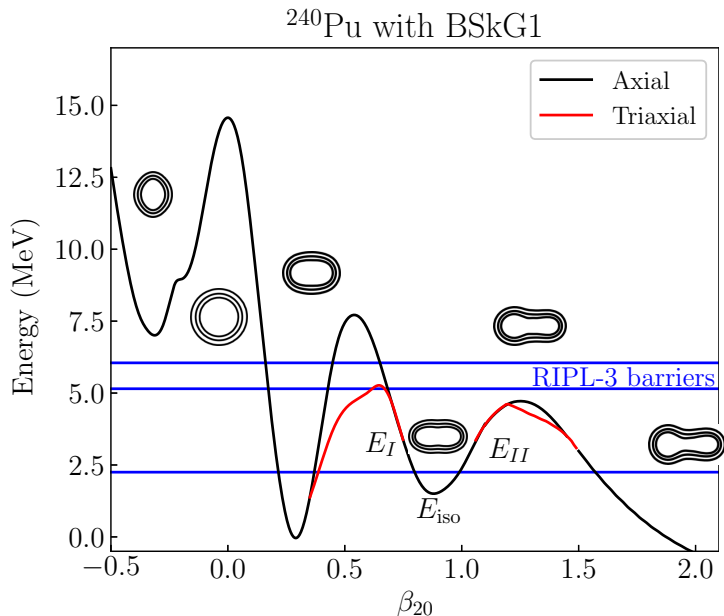
Triaxiality: comparison to experiment



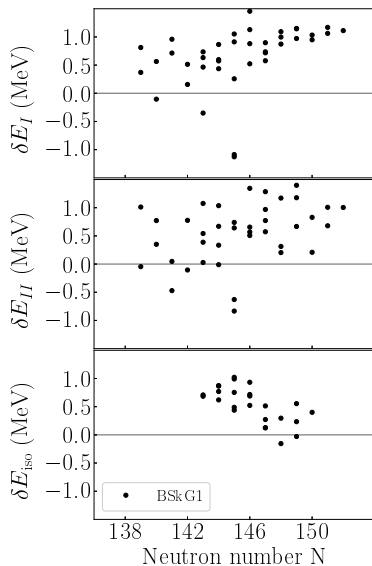
Exp. data from COULEX
extracted by M. Zielińska

- M. Rocchini et al., PRC **103**, 014311 (2021).
- M. Sugawara et al., EPJA, **16**, 409 (2003).
- A. D. Ayangeakaa et al., PLB **754**, 254 (2016).
- Y. Toh et al., EPJA **9**, 353 (2000).
- A. D. Ayangeaka et al., PRL **123**, 102501 (2019).
- E. Clément et al., PRC **75**, 054313 (2007).
- A. E. Kavka et al., NPA **593**, 177 (1995).
- E. Clément et al., PRC **94**, 054326 (2016).
- M. Zielińska, Ph.D. thesis, Warsaw U., (2005).
- M. Zielińska et al., NPA **712**, 3 (2002).
- K. Wrzosek-Lipska et al., PRC **86**, 064305 (2012).
- J. Srebrny et al., NPA **766**, 25 (2006).
- K. Wrzosek-Lipska et al., APB **51**, 789 (2020).
- C. Fahlander et al., NPA **485**, 327 (1988).
- L. E. Svensson et al., NPA **584**, 547 (1995).
- L. Morrison et al., PRC **102**, 054304 (2020).
- C. Y. Wu et al., NPA **607**, 178 (1996).

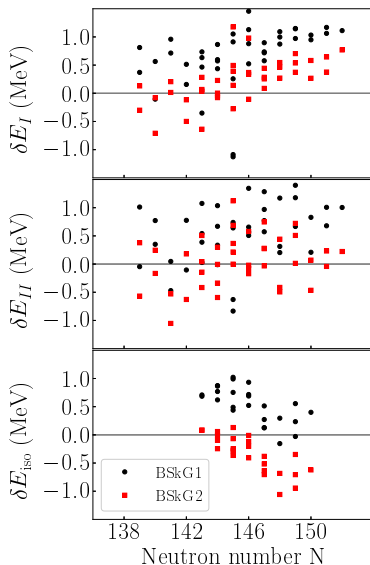
Triaxiality: fission



BSkG2: improved fission barriers above $Z > 90$



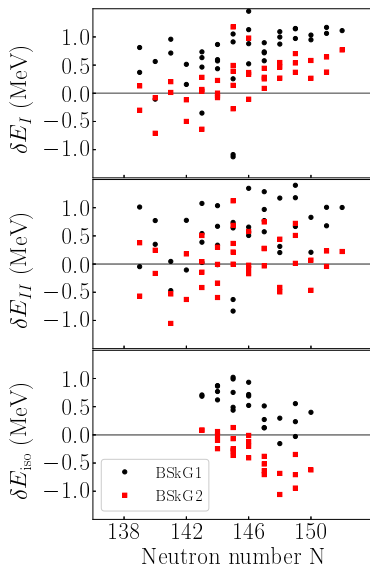
BSkG2: improved fission barriers above $Z > 90$



(MeV)	BSkG1	BSkG2
$\sigma(M)$	0.741	0.668
$\sigma(E_I)$	0.853	0.447
$\sigma(E_{II})$	0.792	0.450
$\sigma(E_{iso})$	0.620	0.465
	HFB-14	FR(L)DM
$\sigma(M)$	0.729	0.560
$\sigma(E_I)$	0.591	0.763
$\sigma(E_{II})$	0.717	n.a.
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HFB-14 from S. Goriely *et al.*, PRC **75**, 064312 (2007).
 FRLDM from P. Möller *et al.*, PRC **79**, 064304 (2009).

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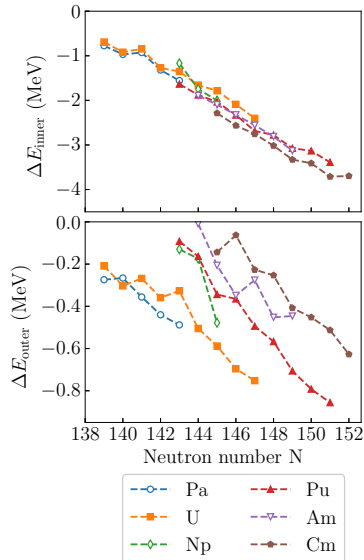


Refitting with **axial** barriers requires **sacrificing** either masses or barriers!

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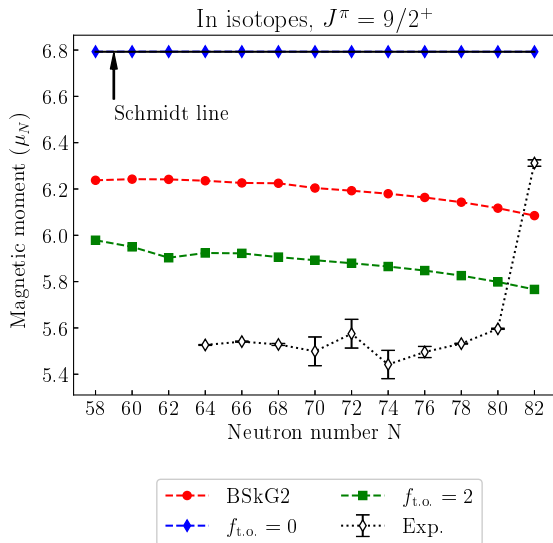


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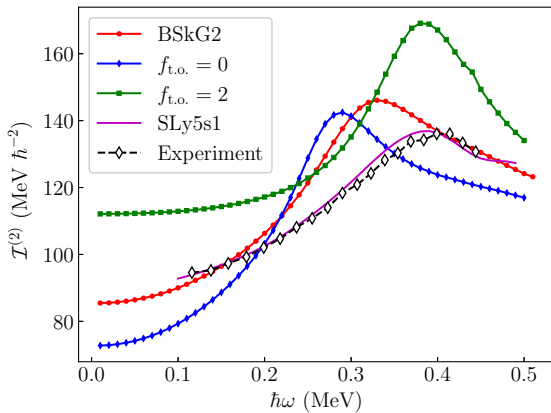
HFB-14 from S. Goriely *et al.*, PRC **75**, 064312 (2007).
FRLDM from P. Möller *et al.*, PRC **79**, 064304 (2009).

Magnetic moments



Exp. data from A. Vernon et al., under review at Nature. DOI:10.21203/rs.3.rs-611360/v1

Superdeformed rotational band in ^{194}Hg



Exp. data from B. Singh, Nuclear Data Sheets 97, 241 (2002).

Conclusions

We are bringing nuclear-state-of-the-art to large-scale modelling.

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Tools

- MOCCa is fast, accurate, robust and versatile
- Machine learning brings MOCCa to the scale of the nuclear chart

Models

- BSkG1: first global model with triaxial deformation
 - competitive on $\sigma(M)$
 - excellent reproduction of COULEX data on triaxial deformation

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Tools

- MOCCa is fast, accurate, robust and versatile
- Machine learning brings MOCCa to the scale of the nuclear chart

Models

- BSkG1: first global model with triaxial deformation
 - competitive on $\sigma(M)$
 - excellent reproduction of COULEX data on triaxial deformation
- BSkG2: first global model with time-odd terms
 - competitive on $\sigma(M)$
 - among the best for actinide fission barriers
 - Analysis ongoing: new observables accessible

Theory questions

- How to improve collective corrections? Linear response?
- Can we finetune the time-odd sector? Should we?
- Can we build continuous fission barriers for odd nuclei?
- Can we use MOCCa for nuclear pasta calculations?
- Can we gain another factor 10 in CPU time through algorithms?

Theory questions

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Model construction

- Extend the fission model
 - Calculate thousands of nuclei with MLNN help
 - including fission yields with scission point model (SPY by J.-F. Lemaître)
 - and half-life calculation with collective inertias
- Develop associated level density model
- BSkG4-5-... will be extended Skyrme forms
 - t_4 & t_5 terms for stable INM and stiff EOS
 - N2LO generalized Skyrme functionals for spectroscopy
 - MOCCa has code-generation for complicated EDFs!

Thanks!

Thanks for...

...the **help**:

BSkG1/2

- S. Goriely
- G. Scamps
- J.-F. Lemaître

ULB

ULB

CEA

MOCCa

- P.-H. Heenen
- M. Bender
- B. Bally

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...the **CPU time**:

- CÉCI network, Belgium
- The Zenobe cluster

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...the **support**:



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...the **CPU time**:

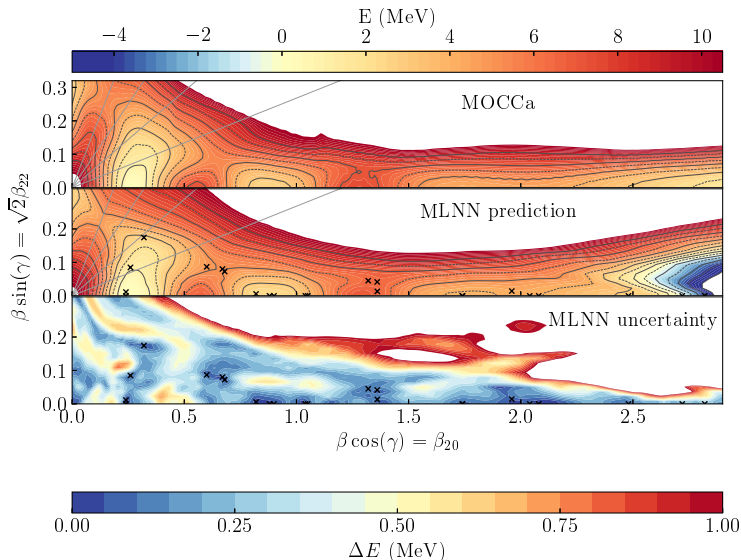
- CÉCI network, Belgium
- The Zenobe cluster

...the **support**:



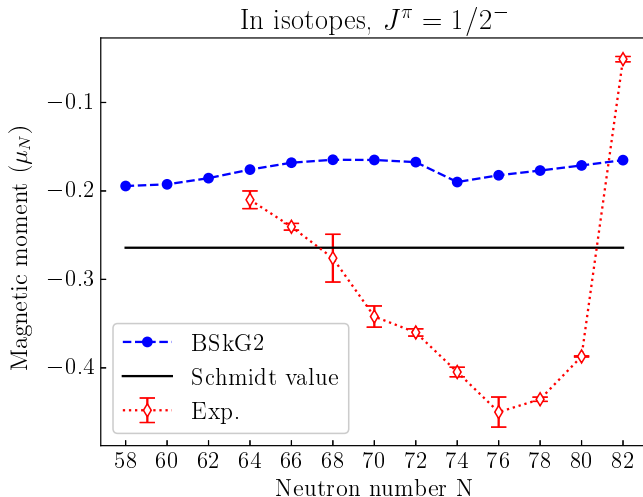
...your **attention**!

A taste of things to come

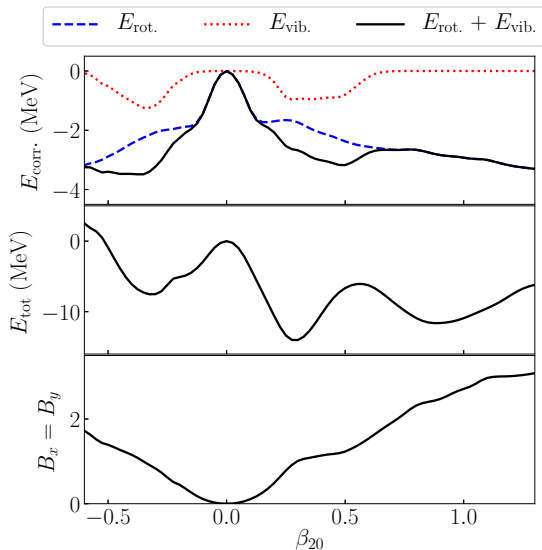


Bonus!

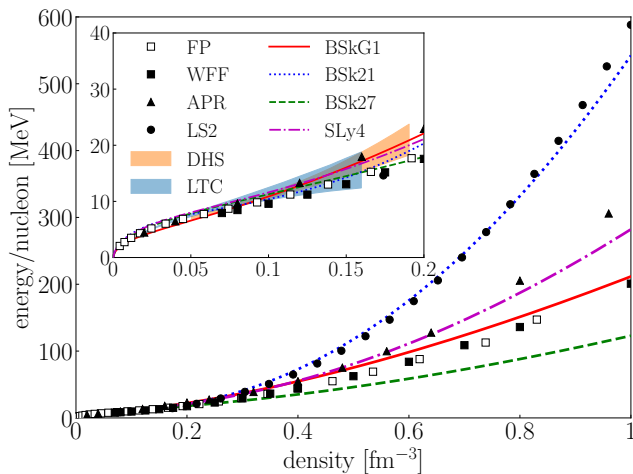
More In isotopes



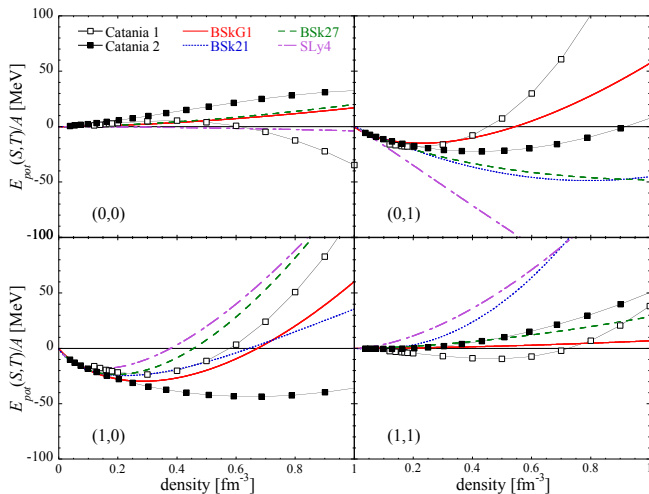
Example of the collective correction: ^{240}Pu



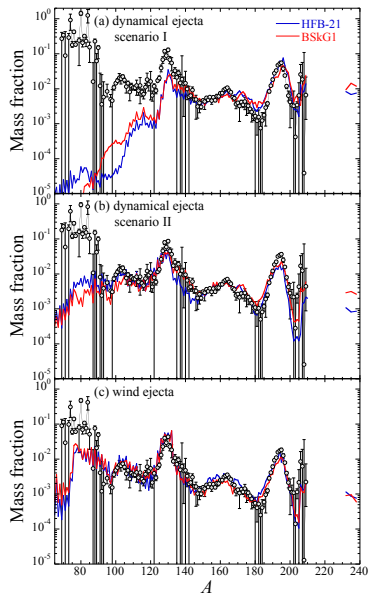
Neutron matter



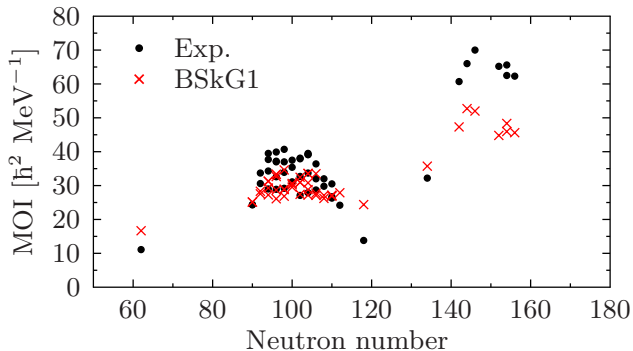
Symmetric matter



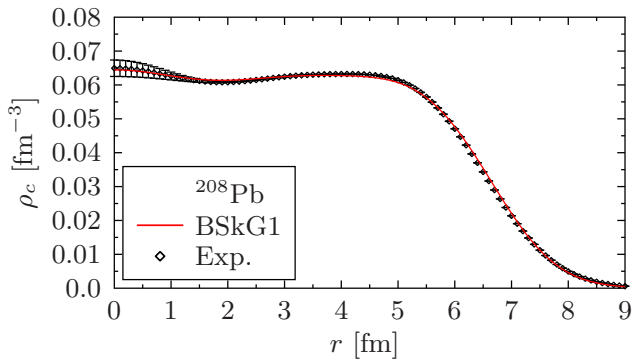
R-process



Moment of inertias



Charge density



Charge radii

