

Broken symmetries in nuclear mean field approaches with Skyrme functionals.

Successes, failures and what we learn from them.

W. Ryssens¹

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29th of August 2018



Mean-field with MOCCa.

Things that work and things that don't.

W. Ryssens¹

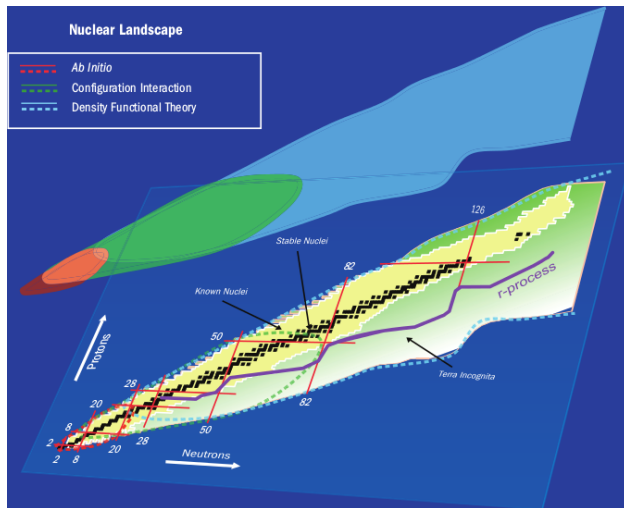
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- 1 Introduction: Nuclear structure with EDFs
- 2 The role of (spatial) symmetries
- 3 Octupole deformation for actinides.
- 4 Hg isotopes
- 5 Rotational bands: Hg, Th and Pr.
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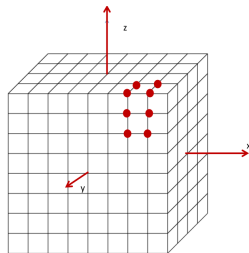
(Outdated) Figure of the UNEDF project from G .F. Bertsch et al., SciDAC Review 6, 48 (2007).

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Variational space → Single-particle states

- represented on a 3D mesh.
- form many-body states ...
- ... as (generalized) product states
- Also determined by **symmetries** !

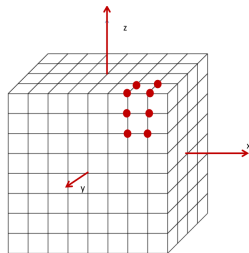


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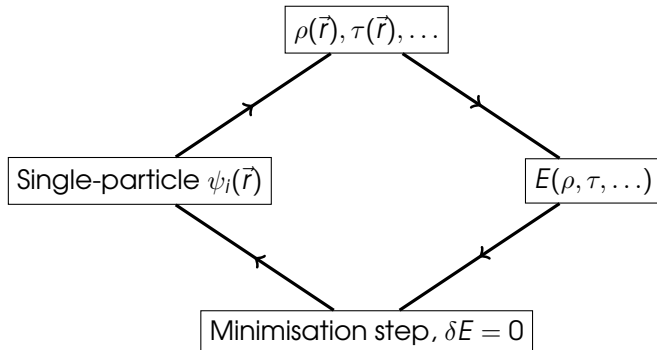
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Products states maintain simplicity, ...
Symmetry breaking to enrich!

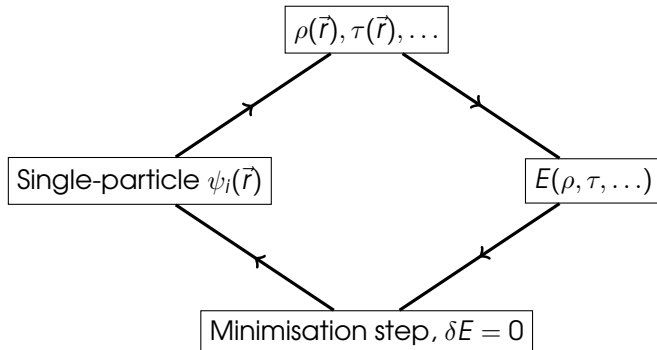


$\Psi_{\text{many-body}}$ = Product wavefunction

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Complexity determined by the single-particle space!

Multiple terms

$$E_{\text{tot}} = E_{\text{kin}} + E_{\text{Skyrme}} + E_{\text{Coul}} + E_{\text{cm}} + E_{\text{pair}},$$

The more or less standard Skyrme part is the following

$$E_{\text{Skyrme}} = \sum_{t=0,1} \int d^3r \left[C_t^\rho \rho_t^2(\vec{r}) + C_t^{\rho^\alpha} \rho_0^\alpha(\vec{r}) \rho_t^2(\vec{r}) + C_t^\tau \rho_t(\vec{r}) \tau_t(\vec{r}) \right. \\ \left. + C_t^{\Delta\rho} \rho_t(\vec{r}) \Delta\rho_t(\vec{r}) + C_t^{\nabla \cdot J} \rho_t(\vec{r}) \nabla \cdot \vec{J}_t(\vec{r}) - C_t^I \sum_{\mu,\nu} J_{t,\mu\nu}(\vec{r}) J_{t,\mu\nu}(\vec{r}) \right].$$

which has

- Densities: $\rho, \tau, J_{\mu\nu}$
- $\sim$ **10** parameters ...
- ... fitted to a few masses and radii.

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Only half of the story;
also time-odd terms!

'How' a system deforms (semi-classical) is a competition

Coulomb repulsion $\longleftrightarrow$ Surface tension

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Coulomb repulsion $\longleftrightarrow$ Surface tension

(MeV)	$a_{\text{surf}}^{(\text{MTF})}$		$a_{\text{surf}}^{(\text{MTF})}$
SLy5s1	18.0	SLy5s5	18.8
SLy5s2	18.2	SLy5s6	19.0
SLy5s3	18.4	SLy5s7	19.2
SLy5s4	18.6	SLy5s8	19.4

Parameterizations from
R. Jodon *et al.*, PRC **94** 024335 (2016).

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- Comparable shell structure
- **Easily** deformable (s1) vs. ...
- **Stiff** deformation (s8).

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Global microscopic framework to describe

Bulk properties

- Masses
- Radii

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Low-energy spectroscopy

- Ground states
 - 0^+ for even-even nuclei
 - J^π for even-odd
- Low-lying excitations
 - Isomers of various kinds
 - Rotational bands

Global microscopic framework to describe

Bulk properties

- Masses
- Radii

Rather good: **HFB17** describes

S.Goriely et al., PRL **102**, 152503 (2009).

- **2149** masses
RMS deviation = **0.581** MeV
Cf. $BE(^{208}\text{Pb}) \sim \mathbf{1.6}$ GeV.
- **792** charge radii
RMS deviation = **0.0313** fm
Cf. $R_{\text{rms}}(^{208}\text{Pb}) = \sim \mathbf{5.5}$ fm.

Low-energy spectroscopy

- Ground states
 0^+ for even-even nuclei
 J^π for even-odd
- Low-lying excitations
 - Isomers of various kinds
 - Rotational bands

This is not easy at all . . .

L. Bonneau et al., PRC **76**, 024320.

- Correct $J^\pi \sim 80\%$ (spherical)
- Correct $J^\pi \sim 40\%$ (deformed)
- Similar performance as MIC-MAC.

The N2LO extension

$$\begin{aligned}
 E_{\text{Skyrme}} &= E_{\text{Skyrme}}^{\text{NLO}} + E_{\text{Skyrme}}^{\text{N2LO}} \\
 E_{\text{Skyrme}}^{\text{N2LO}} &= \sum_{t=0,1} \left[C_t^{\Delta\rho\Delta\rho} (\Delta\rho_t(\vec{r}))^2 + C_t^{\tau\tau} \tau_t^2(\vec{r}) + C_t^{\rho Q} \rho_t(\vec{r}) Q_t(\vec{r}) \right. \\
 &\quad + 2C^{\tau\mu\nu\tau\mu\nu} \sum_{\mu,\nu} \tau_{t,\mu\nu}^2(\vec{r}) - 2C^{\tau\nabla\nabla\rho} \sum_{\mu,\nu} \tau_{t,\mu\nu}(\vec{r}) \nabla_\mu \nabla_\nu \rho_t(\vec{r}) \\
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 \end{aligned}$$

which has

- Densities: Q , $V_{\mu\nu}$, $K_{\mu\nu\kappa}$
- **14** (cf. 10) parameters ...
- ... fitted to a few masses and radii.
- But what new information to include?

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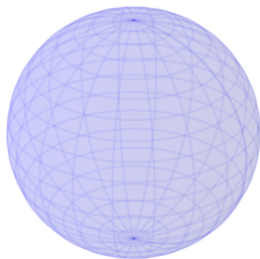
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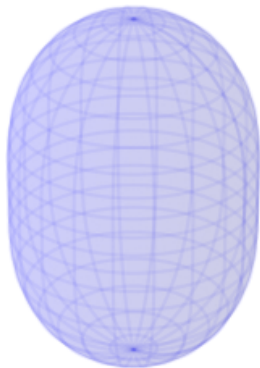
- 1 Introduction: Nuclear structure with EDFs
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- a Ultra-Fast calculations ($\sim$ seconds)
- b Symmetries respected
- c Most EDFs fitted at this level
- d Single-particle quantum numbers J, J_z, π

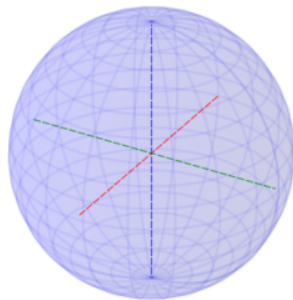


- a Fast calculations ($\sim$ minutes)
- b One symmetry axis
- c Single-particle quantum numbers

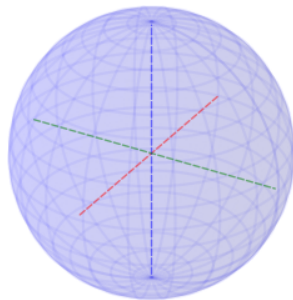
$$K = J_z, \pi$$



- a Slow calculations ($\sim$ hours/days)
- b No more continuous symmetries

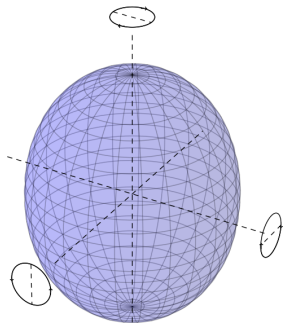


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Single-particle symmetry operators

$$\hat{P}, \hat{R}_{x/y/z}, \hat{T}$$



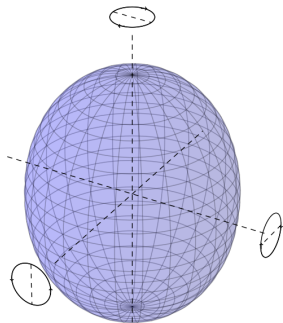
Single-particle symmetry operators

$$\hat{P}, \hat{R}_{x/y/z}, \check{T}$$

give rise to many-body groups

$$\mathcal{D}_{2h}^I = \left\{ \hat{1}, \hat{P}, \check{T}, \check{T}^I, \hat{R}_\mu, \check{R}_\mu^I, \hat{S}_\mu, \check{S}_\mu^I \right\} \text{ (even)}$$

$$\mathcal{D}_{2h}^{ID} = \left\{ \hat{1}, -\hat{1}, \hat{P}, -\hat{P}, \check{T}, -\check{T}, \hat{R}_\mu, -\hat{R}_\mu, \check{R}_\mu^I, -\check{R}_\mu^I, \right. \\ \left. \hat{S}_\mu, -\hat{S}_\mu, \check{S}_\mu^I, -\check{S}_\mu^I, \check{T}^I, -\check{T}^I \right\} \text{ (odd)}$$



All subgroups in principle contain different correlations/physics.

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MOCCa

- * 3D coordinate representation
- * Discrete symmetry breaking
16(!) different combinations
- * Easy to use and fast

W. Ryssens et al., (almost) Submitted to EPJA.

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- * MOCCa v1

NLO functionals

SLy5s1-8 today

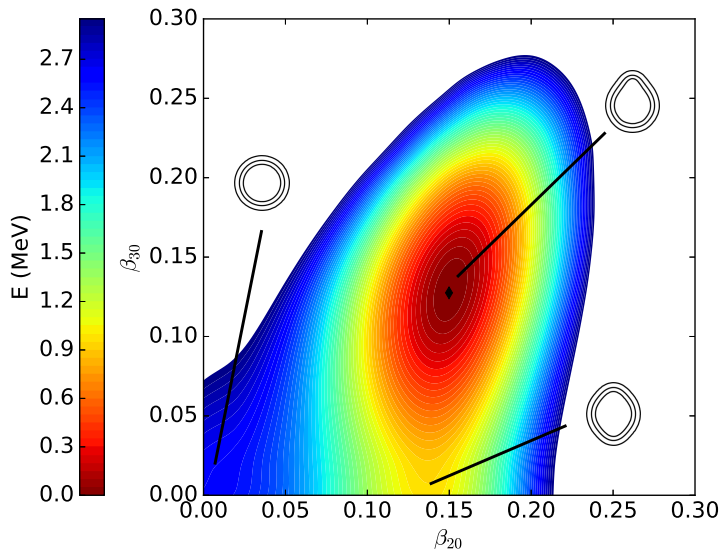
R. Jodon et al. , PRC **94**, 024335 (2016)

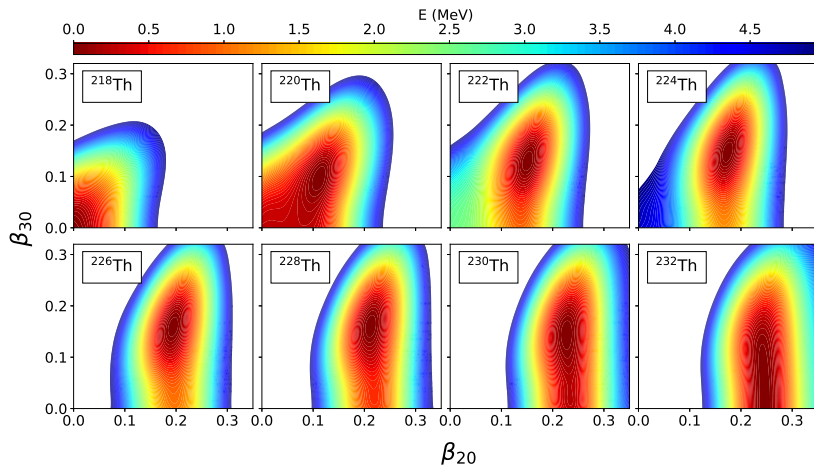
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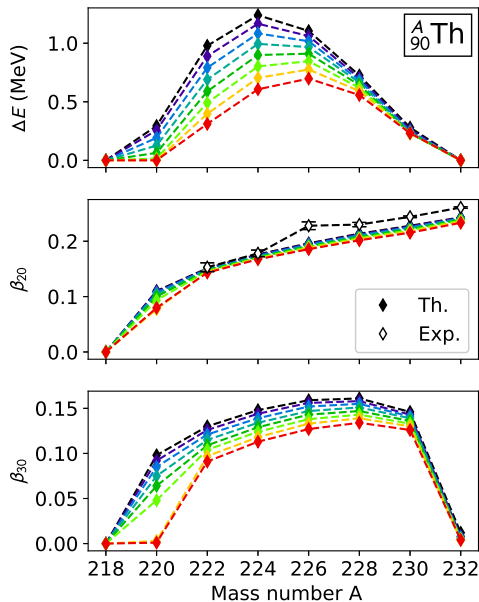
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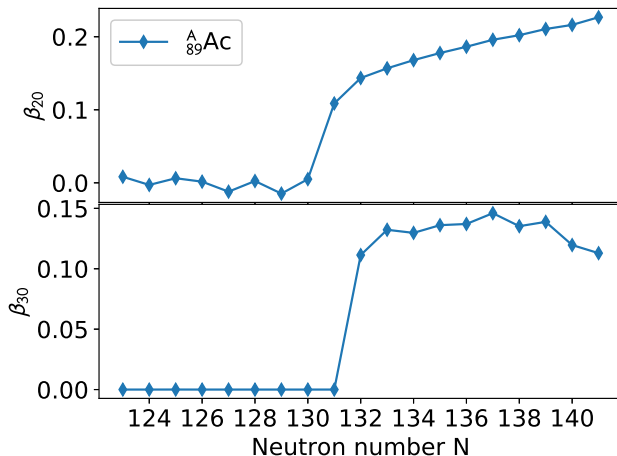
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W. Ryssens et al., Submitted to PRC.
- * MOCCa v2, *Tantalus*
N2LO functionals.
N3LO, N4LO, ...
Automation from formula to code

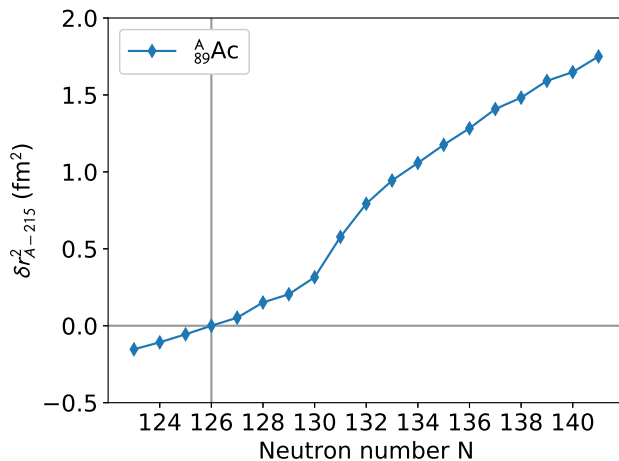
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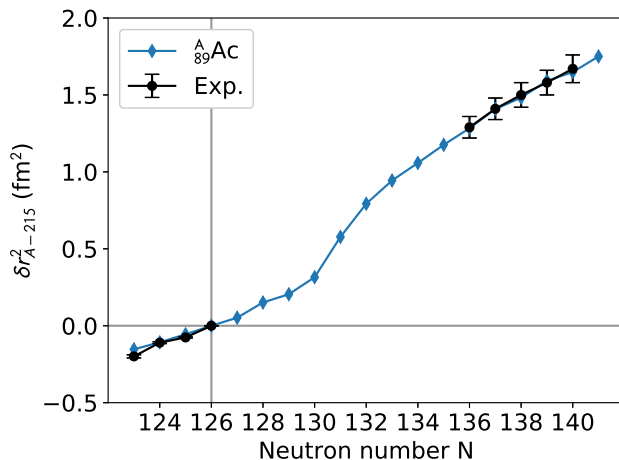








(Preliminary) Data from E. Verstraelen, A. Teigelhoefer et al. (unpublished).



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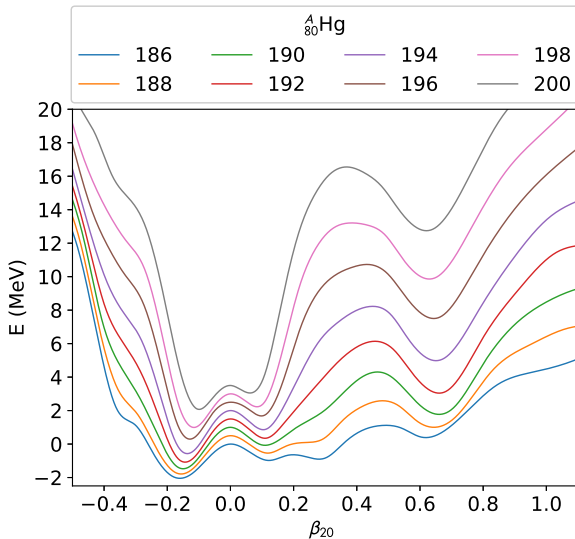
Odd Th isotopes

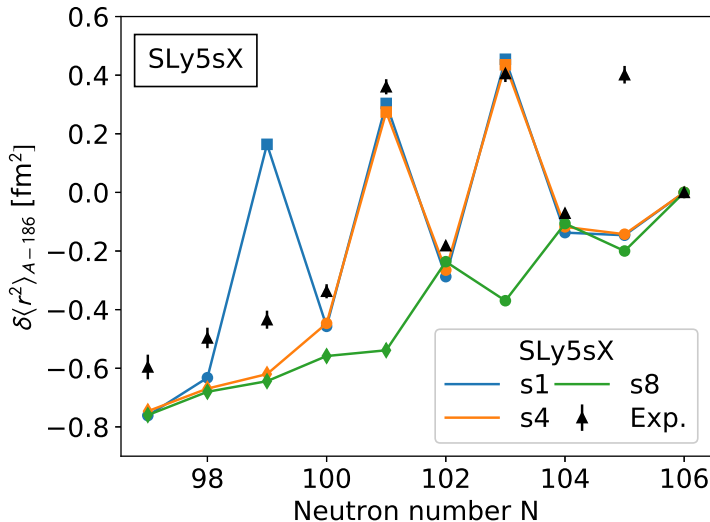
A	Exp.	Th.
221	$7/2^+$	$1/2$
223	$5/2^+$	$5/2$
225	$3/2^+$	$3/2$
227	$1/2^+$	$1/2$
229	$5/2^+$	$1/2$
231	$5/2^+$	$1/2$

Ac isotopes

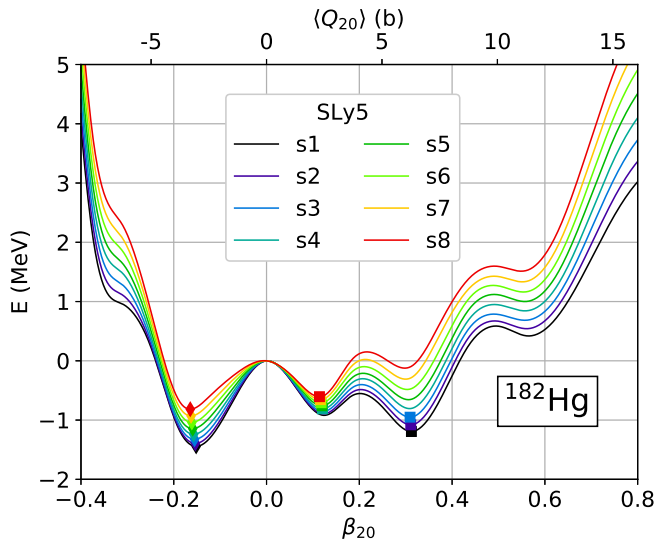
A	Exp.	Th.
212	(7)	(4)
213	(9/2)	$1/2$
214	5	(4)
215	$9/2$	$5/2$
225	$3/2$	$3/2$
226	(1)	(2)
227	$3/2$	$3/2$
228	3	(2)
229	$3/2$	$3/2$

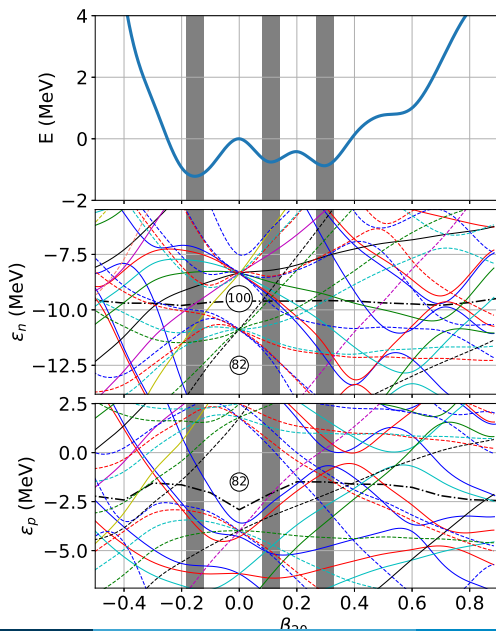
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S. Sels et al., (almost) submitted to PRC.





A	Exp.	SLy5s1	SLy5s8.
177	$(7)/2^-$	$1/2^-$	$1/2^-$
179	$(7)/2^-$	$1/2^-$	$1/2^-$
181	$1/2^-$	$5/2^-$	$1/2^-$
183	$1/2^-$	$5/2^-$	$3/2^-$
185	$1/2^-$	$3/2^-$	$3/2^-$

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Modelling rotation with
Skyrme-HFB cranking Routhian.

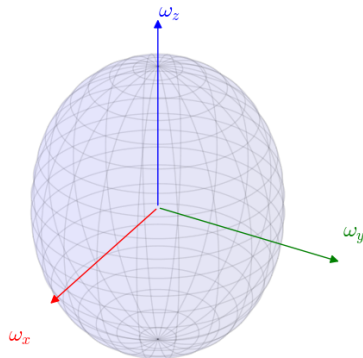
$$R = E - \omega_x \langle \hat{J}_x \rangle - \omega_y \langle \hat{J}_y \rangle - \omega_z \langle \hat{J}_z \rangle$$

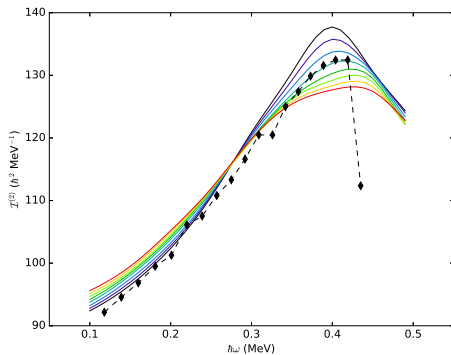
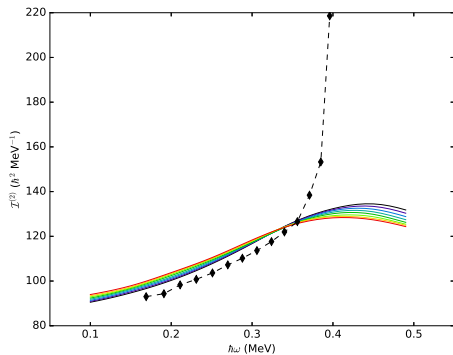
Preferred directions $\Rightarrow$ signatures

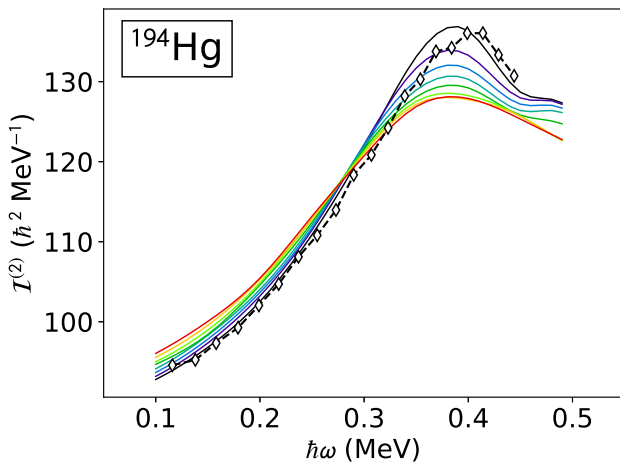
Compared to experiment

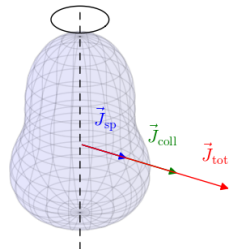
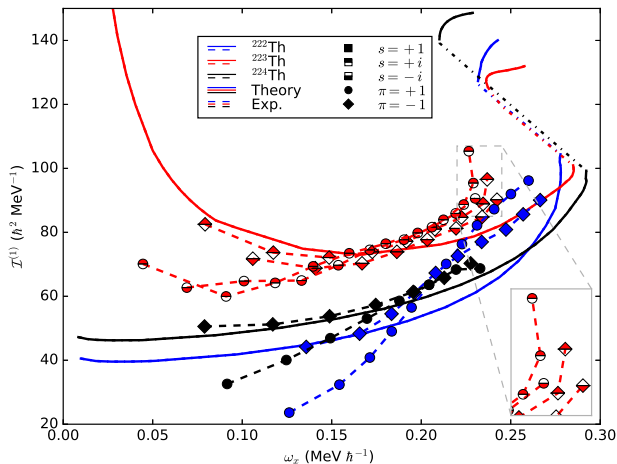
$$\omega(J) = \frac{E(J+1) - E(J-1)}{I(J+1) - I(J-1)}.$$

$$I(J) = \sqrt{J^2 - J_{\text{sp}}^2}$$

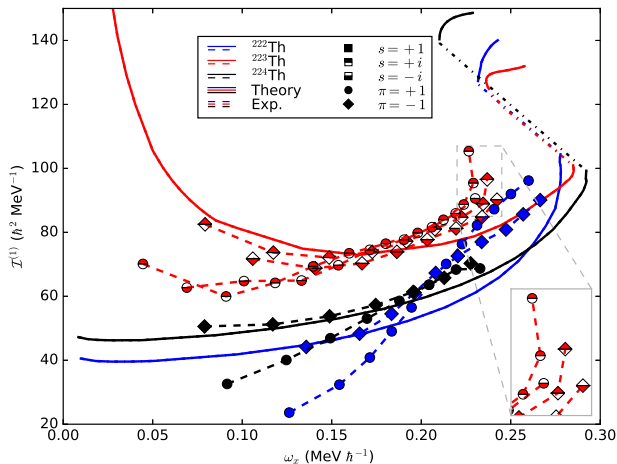




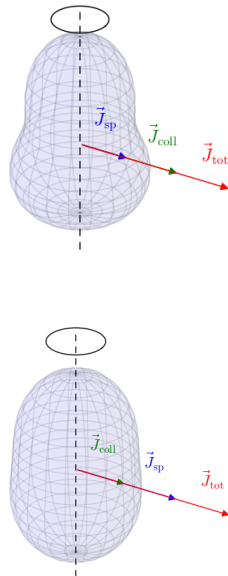


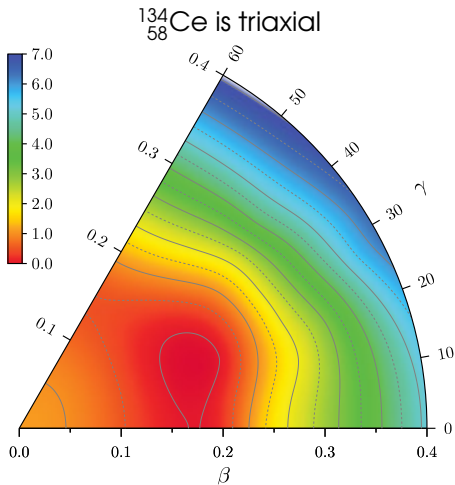


Data from
 G. Maquart *et al.*, Phys. Rev. C **95**, 034304 (2017),
 J.F. Smith *et al.*, Phys. Rev. Lett. **75**, 1050-1053 (1995).

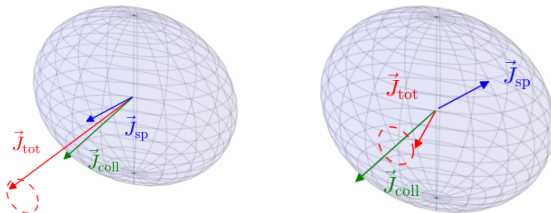


Data from
 G. Maquart *et al.*, Phys. Rev. C **95**, 034304 (2017),
 J.F. Smith *et al.*, Phys. Rev. Lett. **75**, 1050-1053 (1995).

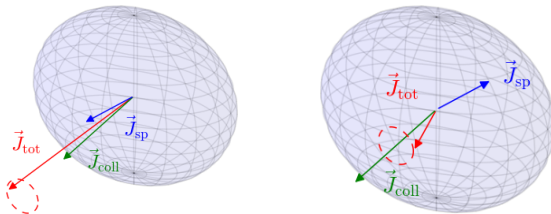




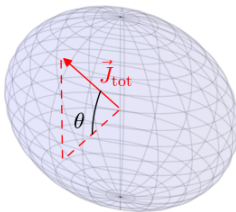
Two ways of combining rotation with an odd particle

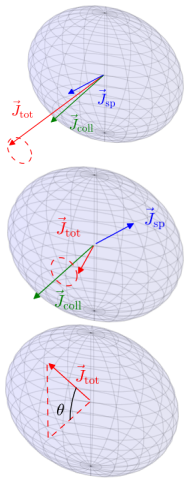


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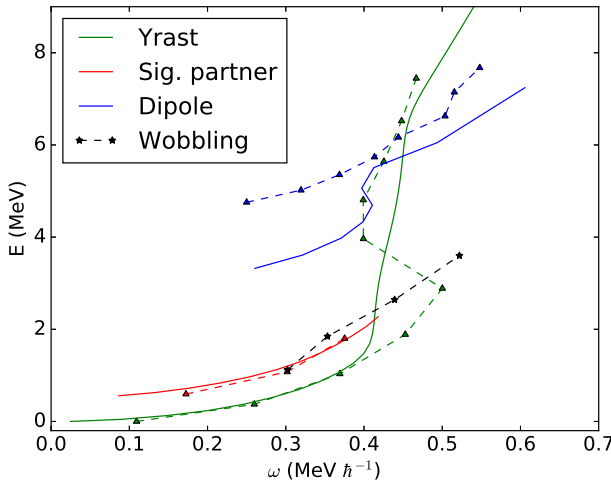


But on top of that options to spin about non-principal axes





Exp. data from
PRL **114**, 082501 (2015).



- 1 Introduction: Nuclear structure with EDFs
- 2 The role of (spatial) symmetries
- 3 Octupole deformation for actinides.
- 4 Hg isotopes
- 5 Rotational bands: Hg, Th and Pr.
- 6 Conclusion

Nuclear structure calculations with EDFs:

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 - Future projects: tetrahedral deformations, 2-q β bands, ...
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 - Charge radii systematics, backbending in bands, ...
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- Surface tension → Control certain aspects of functionals.
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Thanks for the CPU time! (for W.R.)

CC of the IN2P3

CECI network

CNRS, France

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